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VOLUME 7

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NUMBER 1

EFFECT OF RATE OF SEEDING UPON COMPARISON OF VARIETIES OF OATS¹

BY J. W. HOPKINS²

Abstract

Three varieties of oats: Abundance, Banner and Daubeney, were sown at various rates in 1924-25-26 at four places differing widely in soil and climate, *vis.*, Charlottetown, P.E.I., Macdonald College, Que., Scott, Sask., and Edmonton, Alberta. Daubeney has smaller seed and tillers more freely than the other two varieties.

The method of field experimentation adopted is considered in the light of the principles of "randomisation" and "local control" developed at Rothamsted by R. A. Fisher. The plots sown at each rate were adequately replicated but their systematic arrangement precluded a valid estimate of the errors in yield comparisons. In particular, the arrangement of the plots sown at different rates was shown to coincide to some extent with variations in soil fertility.

In general the differences in yield between plots of the same variety sown at different rates were small. Daubeney showed the greatest variations; the expected stabilizing effect of its tendency to tiller did not appear.

The combined results of the three years indicate that the optimum rate of seeding may not be the same for different varieties at the same station, and certainly is not the same for the same variety at different stations. Near the optimum the effect of variations in seed rate upon yield were slight and, at three out of the four stations, all three varieties might be sown at a specified uniform rate without significantly increasing the ordinary experimental error.

Large fluctuations were found in the percentage stand of plants recorded, due in part to unavoidable errors in counting, but still indicating significant differences between the stations. At three stations the optimum stand of Daubeney may have been higher than that of the other two varieties, though at only one station is the difference significant.

1. Introductory

The important place occupied in Canadian agricultural science by that type of field experimentation known as "variety trials" is well known. Not only are such trials essential in determining the suitability, to any particular set of environmental conditions, of the very many strains and varieties of all important crops which now exist, they are also the only means at our

¹ Manuscript received May 23, 1932.

A statistical study of experimental data accumulated by the Associate Committee on Accurate Plot Work during the seasons 1924-25-26. The membership of the Committee and brief descriptions of the co-operative work accomplished are given in the Annual Reports of the President of the National Research Council of Canada for the years ending March 31, 1925-26-27.

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disposal for discovering the true merits of those productions of the geneticist which have played so prominent a part in the extension of agriculture to regions hitherto held unsuitable, or, less spectacular but equally important, have materially increased the productivity of areas already under cultivation. There can be few agricultural experiment stations in the country not at the present time actively engaged in field trials of this nature.

Notwithstanding the extent to which such experimentation has been and is being carried on, some uncertainty has existed as to the exact procedure by which the desired result, namely, the unbiased comparison of the productive capacity of different strains of the same crop, might best be attained. As a result, there has been considerable divergence of practice (2). It was in view of the obvious and important advantages accruing from any increase in uniformity of procedure and accuracy that the present investigation, which represents an attempt to resolve some at least of the points at issue, was undertaken.

The difficulties here investigated centre around the fact that the varieties to be compared usually differ in many other characters besides inherent yielding capacity. One of the most obvious of these is size of seed.

Size of seed, or weight per 1000 kernels, which is the quantitative measure usually adopted, is known to vary considerably from variety to variety. It also varies from sample to sample of the same variety, depending upon the soil and climatic conditions under which the seed was produced. In consequence, the Swedish experimental stations have long adopted the principle, which also has its adherents in Canada (13), that in comparative tests equal numbers of germinable seeds should be applied to equal areas of land.

This principle has never gained any wide acceptance on this continent, where the common practice is to sow equal *weights*, rather than equal numbers of seeds. Experience here also indicates that the Swedish procedure does not sufficiently take into account two facts: that a really fair comparison of productive capacity is possible only when each variety is seeded at its *optimum* rate, and that this optimum rate may be influenced by size of seed as well as by other factors.

For example, a large number of experiments have been performed to test the yielding capacity of large and small seed of the same variety. Summarizing all such data then available dealing with small grain crops Kiesselbach (12) found that, when both were planted in equal numbers at a rate optimum for the large seed, the small seed yielded 11% less grain per acre than the large. When equal weights were planted, however, again at a rate optimum for the large seed, the small seed yielded only 3% less. It would thus appear that, quite apart from the varietal factor, small seeds should be planted in greater numbers than large if they are not to be placed at an unfair disadvantage.

The fact that different varieties may have considerably different optimum seed rates has been observed in the case of corn, wheat and oats. Mooers (14), speaking of corn, says, "On the same land different varieties of similar length of season and habit of growth may differ appreciably in the rate of planting which gives best results". Osborn (15), also working with corn, noted certain

varietal differences in response to increased seed rate. He found that the optimum rate of seeding was influenced by the fertility of the soil, and gave it as his opinion that no specified rate of planting of the same variety could be depended on to produce maximum yields in successive seasons even on the same character of land, owing to changed climatic conditions. Grantham (8) found considerable differences in the tillering capacity of wheat varieties. When varieties differing in this respect were sown at three different rates (equal numbers of seeds of the eight varieties being sown in each case), marked varietal differences were found in response to increase in rate of seeding. Stanton *et al.* (16) have carried out a rate of seeding experiment with fall-sown oats at Athens, Ga. Two varieties, Red Rustproof and Fulghum, were used. The optimum rate for Red Rustproof was found to be six pecks, whereas the yield of Fulghum increased progressively with the seed rate up to a rate of 10 pecks, the highest used, thus indicating that the optimum *weight* of seed may be no more constant than the optimum number.

We therefore perceive that the optimum stand of any crop under a given set of conditions may be influenced by a number of factors—morphological, physiological, and environmental—which may be expected to interact in a complicated and often obscure manner. In consequence it would appear that an investigation into the optimum stand of crops under different conditions of soil and climate might do much to put variety testing on a sounder scientific basis.

In 1923 the National Research Council of Canada appointed an Associate Committee on Accurate Plot Work, with representatives of the Federal Department of Agriculture and of various agricultural colleges. Certain members of this committee drew up a scheme of investigation into the effects of rates of seeding upon the comparison of varieties of oats, the experimental work being carried out during the seasons 1924, 1925 and 1926. The Dominion Cerealists, as Secretary of the Committee, collected and filed each year the experimental data thus obtained; these form the basis of the present report.

2. Experimental Procedure

In addition to its importance in Canadian agriculture, oats had certain obvious advantages for the purposes of this experiment. Those variations in seed size, the effect of which it was desired to study, are here pronounced, and in addition there are often marked differences between varieties in respect of tendency to tiller.

To achieve the objects of the investigation it appeared that experimentation should be conducted at a number of places in different parts of the country. This should enable the effect of variations in soil and climatic conditions to be assessed and conclusions of some generality to be obtained. Furthermore it was recognized that too much reliance could not be placed on the results of a single season's work. It was therefore decided that the experiment should be carried on for a period of three years.

Three varieties of oats were selected for experimentation: Abundance, a variety supposedly characterized by fairly large seed and low tendency to

tiller; Banner, a variety medium in size of seed and tendency to tiller; and Daubeney, a small-seeded variety with pronounced tillering propensities.

The general plan supplied to all co-operating stations showed the plots laid out in three long ranges, one range being devoted to each of the experimental varieties. In practice, each station adapted the scheme to its local field arrangement. At the University of Alberta, for example, the experimental field is laid out in blocks about eight rods wide. The planting of the oat plots was begun at one corner of such a block and carried back and forth alternately across this distance until one variety was completed, when the next variety was immediately begun. The whole experiment was thus contained within an area about 8 by 11 rods in size.

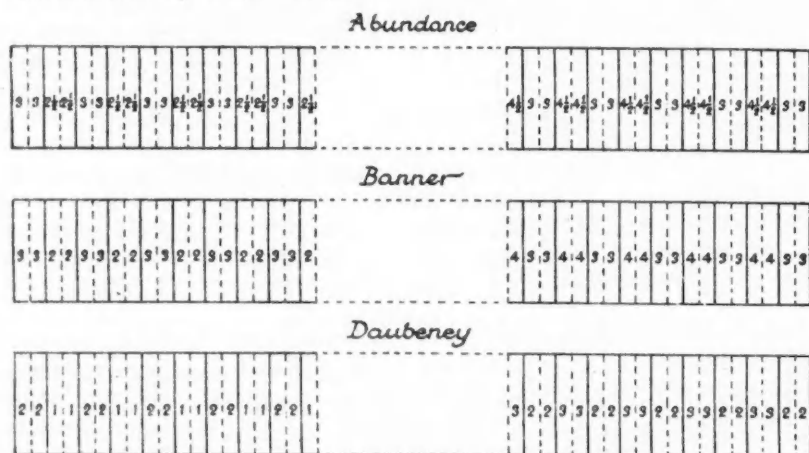


FIG. 1. Scheme of arrangement of plots in the field.

Each plot consisted of nine drills $18\frac{1}{2}$ ft. long. At harvest time one foot was removed from each end of the plot to eliminate "border effect", the plot actually harvested being thus exactly one rod in length. The drills were spaced 7 in. apart, except at Macdonald College, where one link was the spacing adopted. In addition to the removal of one foot from each end of the plot, the two outside rows were also discarded at harvest time. Of the remaining seven rows, the central one was dug up in its entirety and the number of plants and tillers contained therein determined; no other use was made of this row. The remaining three rows on either side of this central row were harvested separately in the form of two three-row plots. Each rate of seeding (of which there were four in the case of each variety) occupied eight such nine-row plots, or sixteen of the three-row plots.

The allocation of the plots to the various rates of seeding proceeded in a systematic manner, the eight plots bearing the lowest rate of seeding being grouped together at one end of the block, then the eight plots of the next highest rate, and so on. "Check plots", sown at a rate constant within each

variety, were however alternated with the rate plots, so that each three-row "rate plot" was immediately adjacent to a "check plot" of similar size; it being believed that by utilizing the *differences* in yield between such adjacent plots, rather than the actual yields themselves, the precision of the experiment would be enhanced. The whole experiment was in fact laid out with the object of obtaining a series of differences in yield between adjacent "check" and "rate" plots, the significance of which could be tested by the well-known method first propounded by "Student".

The total number of large, or nine-row, plots in each "range" amounted to 65, allowing for a check plot at each end. The general plan of arrangement is illustrated diagrammatically in Fig. 1. The complete series of rates, in the order in which they occurred, was as follows:

Range 1. Banner				Range 2. Daubeney				Range 3. Abundance			
Plot	Rate, bu. per acre	Plot	Rate, bu. per acre	Plot	Rate, bu. per acre	Plot	Rate, bu. per acre	Plot	Rate, bu. per acre	Plot	Rate, bu. per acre
1	3.0	34	3.5	65	2.0	98	2.5	129	3.0	162	4.0
2	2.0	35	3.0	66	1.0	99	2.0	130	2.5	163	3.0
3	3.0	36	3.5	67	2.0	100	2.5	131	3.0	164	4.0
4	2.0	37	3.0	68	1.0	101	2.0	132	2.5	165	3.0
5	3.0	38	3.5	69	2.0	102	2.5	133	3.0	166	4.0
6	2.0	39	3.0	70	1.0	103	2.0	134	2.5	167	3.0
7	3.0	40	3.5	71	2.0	104	2.5	135	3.0	168	4.0
8	2.0	41	3.0	72	1.0	105	2.0	136	2.5	169	3.0
9	3.0	42	3.5	73	2.0	106	2.5	137	3.0	170	4.0
10	2.0	43	3.0	74	1.0	107	2.0	138	2.5	171	3.0
11	3.0	44	3.5	75	2.0	108	2.5	139	3.0	172	4.0
12	2.0	45	3.0	76	1.0	109	2.0	140	2.5	173	3.0
13	3.0	46	3.5	77	2.0	110	2.5	141	3.0	174	4.0
14	2.0	47	3.0	78	1.0	111	2.0	142	2.5	175	3.0
15	3.0	48	3.5	79	2.0	112	2.5	143	3.0	176	4.0
16	2.0	49	3.0	80	1.0	113	2.0	144	2.5	177	3.0
17	3.0	50	4.0	81	2.0	114	3.0	145	3.0	178	4.5
18	2.5	51	3.0	82	1.5	115	2.0	146	3.5	179	3.0
19	3.0	52	4.0	83	2.0	116	3.0	147	3.0	180	4.5
20	2.5	53	3.0	84	1.5	117	2.0	148	3.5	181	3.0
21	3.0	54	4.0	85	2.0	118	3.0	149	3.0	182	4.5
22	2.5	55	3.0	86	1.5	119	2.0	150	3.5	183	3.0
23	3.0	56	4.0	87	2.0	120	3.0	151	3.0	184	4.5
24	2.5	57	3.0	88	1.5	121	2.0	152	3.5	185	3.0
25	3.0	58	4.0	89	2.0	122	3.0	153	3.0	186	4.5
26	2.5	59	3.0	90	1.5	123	2.0	154	3.5	187	3.0
27	3.0	60	4.0	91	2.0	124	3.0	155	3.0	188	4.5
28	2.5	61	3.0	92	1.5	125	2.0	156	3.5	189	3.0
29	3.0	62	4.0	93	2.0	126	3.0	157	3.0	190	4.5
30	2.5	63	3.0	94	1.5	127	2.0	158	3.5	191	3.0
31	3.0	64	4.0	95	2.0	128	3.0	159	3.0	192	4.5
32	2.5	64a	3.0	96	1.5	128a	2.0	160	3.5	193	3.0
33	3.0			97	2.0			161	3.0		

All plots were sown by hand, a definite *weight* of seed being placed in each drill. All seed used was of common origin, being supplied by the Dominion Cerealists, Central Experimental Farm, Ottawa. Information regarding the weight per 1000 kernels and percentage germination was also supplied for the

seed of each variety each year. These values, however, were checked in most cases by the individual stations, at the request of the Dominion Cerealists, and in some cases where the divergence of results was appreciable, the stations used their own figures in calculating the number of germinable seeds sown in each plot.

The experimental work was commenced in 1924. An invitation to participate was issued to a number of experimental stations and agricultural colleges, at the following eight of which the experiment was actually carried out during the first season:

Experimental Station, Charlottetown, P.E.I.

Experimental Station, Cap Rouge, Quebec.

Macdonald College, Quebec.

Central Experimental Farm, Ottawa, Ont.

University of Manitoba, Winnipeg, Man.

Experimental Station, Scott, Sask.

University of Alberta, Edmonton, Alberta.

University of British Columbia, Vancouver, B.C.

Owing to a variety of causes the number of the co-operating stations progressively declined during the two remaining seasons of the experiment. The present report is confined to the analysis of the data secured at four stations, *vis.*: University of Alberta, Scott Experimental Station, Macdonald College, and Charlottetown Experimental Station. Fortunately, these stations are very widely separated and represent fairly well the maximum range of soil and climatic conditions likely to be met with in field experimentation in Canada. At all four stations the field operations would appear to have been carried out with all reasonable care, including, at Scott, the surrounding of the area occupied by the plots with wire netting, to prevent the augmentation of the experimental error by the molestations of gophers and rabbits. It is necessary to note, however, that at Macdonald College the variety Abundance was not grown in 1925, and that at Scott in the same year a departure from the general practice in determining the stand of plants and the number of tillers occurred. This took the form of counting the stand of plants in the centre row of each plot in the spring, as soon after emergence as possible. At harvest time, a further count of the total number of culms was made, the difference between these two counts furnishing the estimate of tillering. Whilst any departure from uniformity of procedure would have been regrettable, this one proved especially so owing to the fact that by no means all the plants which emerged survived to maturity. In consequence, no weight can be attached to any estimate of tillering thus derived. Indeed, in some cases the total number of culms counted in the autumn was less than the number of plants counted in the spring.

3. The Principles of Field Experimentation Developed at Rothamsted by R. A. Fisher

It is of course now very widely recognized amongst all thoughtful persons engaged therein, that field trials, in common with most other forms of experimentation, particularly those of a biological nature, should be at least duplicated

and preferably further replicated. The precise nature of the benefits to be derived from this procedure is not perhaps so generally realized. These are in fact twofold. In the first place, since the mean of several determinations is more reliable than a single determination alone, replication increases the *precision* of our results. Secondly, by an examination of the variability of replicate observations, we are enabled to arrive at an estimate of the *experimental error* to which the results are subject. It is only by considering any observed differences in the light of this error that we may arrive at conclusions of any scientific value. The peculiarity of field experimentation lies in that, whilst the second of these considerations, the estimation of error, is here of particular importance, yet the methods adopted have often not been such as to ensure the validity of any such estimate obtained.

This condition of affairs is due to the fact, suggested by a wealth of observational evidence and verified in all careful uniformity trials (9), that any area of land chosen for experimental work may be assumed to be more or less heterogeneous from the point of view of fertility. Moreover this variation, although often extremely complex, nevertheless usually exhibits a systematic element, in the sense that if the whole area were divided into a large number of sub-areas, the fertility of neighboring sub-areas would tend to be positively correlated. In short we have what "Student" has termed "a sort of regular irregularity". It appears that in most agricultural experiments soil heterogeneity is by far the most potent cause of variation in the yields of similarly treated plots. By the exercise of reasonable care the effect of errors in seeding, harvesting, weighing, etc., may be reduced to quite secondary proportions.

From these considerations it is obvious that all experimental design should have two objects. The first is to minimize the disturbing effect of soil heterogeneity upon the comparisons which it is desired to make. The second is to render possible a valid estimate of the errors to which our comparisons, as performed, are subject.

Now the requirement of a valid estimate of error is that the differences in fertility between parallel plots (*i.e.*, plots treated alike) should be representative of the differences in fertility between plots treated differently. It is clear that we have no right to assume that we have achieved this desideratum if any systematic arrangement of plots is superimposed upon land of which the fertility varies from point to point in a manner which also has in it a systematic element. The two systems may well have features in common, in which case our estimate of error will not be of valid application. The direct way to overcome this difficulty according to Fisher (4, 5, p. 229 *et seq.*), is to arrange the plots wholly at random. If this is done, the error, as estimated from plots treated alike, will be a valid estimate of the error affecting comparisons between plots treated differently. The principle of *randomness* should therefore be regarded as an essential condition which any plot arrangement must fulfil.

It is not of course contended that the estimate of error obtained from any random arrangement necessarily coincides with the true error of the experiment. If it did, it would be more than an estimate. But if we imagine the experiment to be performed, under identical soil and climatic conditions,

using all the equally possible random arrangements, then it would be found that the estimates of error thus obtained were clustered around a central value, this central value being the true error of the experiment. It is upon precisely

$4\frac{1}{2}$	$3\frac{1}{2}$	3	4	$2\frac{1}{2}$	2
$3\frac{1}{2}$	$2\frac{1}{2}$	4	3	2	$4\frac{1}{2}$
2	3	$2\frac{1}{2}$	$4\frac{1}{2}$	$3\frac{1}{2}$	4
$2\frac{1}{2}$	4	2	$3\frac{1}{2}$	$4\frac{1}{2}$	3
3	$4\frac{1}{2}$	$3\frac{1}{2}$	2	4	$2\frac{1}{2}$
4	2	$4\frac{1}{2}$	$2\frac{1}{2}$	3	$3\frac{1}{2}$

this expected variation in our estimate, the nature of which can be predicted when the errors by which the observed values are affected are a true random sample of the errors which contribute to our estimate, that the test of significance employed, namely the Z test of Fisher (5, p.190 *et seq.*) is based.

The random arrangement of the plots ensures that the estimate of error obtained is a valid one. It is however possible to utilize the fact, already noted, that the yield of neighboring plots is usually positively correlated, to eliminate from our comparisons certain major elements of soil heterogeneity. The result is a much reduced, but still accurately estimated, experimental error. This is accomplished by imposing certain restrictions upon the purely random arrangement. According to the particular restrictions employed, two types of arrangement are obtained, which have been designated by Fisher "Randomised Blocks" and the "Latin Square", respectively.

$4\frac{1}{2}$	$3\frac{1}{2}$	3	4	$2\frac{1}{2}$	2
D A B A B D	B D A D B A	D B A D B A	B D A D B A	B D A D B A	B D A D B A
$3\frac{1}{2}$	$2\frac{1}{2}$	4	3	2	$4\frac{1}{2}$
D B A D B A	D B A D B A	D B A D B A	D B A D B A	D B A D B A	D B A D B A
2	3	$2\frac{1}{2}$	$4\frac{1}{2}$	$3\frac{1}{2}$	4
B D A A B D	A B D A B D	B D A A B D	B D A A B D	D B A A D B	A D B A A D B
$2\frac{1}{2}$	4	2	$3\frac{1}{2}$	$4\frac{1}{2}$	3
B D A B A D	B A D B A D	B D A B A D	B D A B A D	D A B D D A B	D A B D D A B
3	$4\frac{1}{2}$	$3\frac{1}{2}$	2	4	$2\frac{1}{2}$
A B D B D A	A B D B D A	A B D B D A	B A D A D B	A D B D B A	D B A D B A
4	2	$4\frac{1}{2}$	$2\frac{1}{2}$	3	$3\frac{1}{2}$
D A B A D B	A B D B D A	B D A A B D	B D A A B D	A D B A A D B	A B D A B D

FIG. 2. Above: Random distribution of rates of seeding in a 6 by 6 Latin square. Below: Completed random arrangement for three varieties. Rates of seeding in bu. per acre. A = Abundance, B = Banner, D = Daubeney.

In the former, the experimental area is divided into a number of "blocks" of land, there being as many such blocks as there are to be replications. Each block consists of as many plots as there are treatments, varieties, etc., to be compared. The plots are then allotted at random to the various treatments, subject to the restriction that each treatment may occur once, and once only, in each block. By this arrangement the variations in soil fertility which may

affect our comparisons are confined to such as occur *within* blocks; the random arrangement of the plots in each block allowing the error thus introduced to be properly estimated. Differences in general fertility level between blocks are thus eliminated in the field, and, by the statistical procedure known as the Analysis of Variance (5, p. 190 *et seq.*) may also be eliminated from our estimate of error. There thus often results an experiment of considerably enhanced precision.

A further extension of the principle of restricted random arrangement, or Local Control as Fisher has termed it, leads to the Latin Square. Here there are as many replicates as there are treatments, the plots being arranged in the form of a square. The various treatments are again distributed at random over the plots, subject however to the restriction that each treatment shall occur once only in each horizontal row and each vertical column of plots forming the square. (An illustration of such an arrangement appears in the upper part of Fig. 2). In consequence, it is possible to eliminate soil differences both between rows and between columns of plots (*i.e.*, in two directions at right angles). At the same time a valid estimate of the remaining errors is obtained. When the number of comparisons to be made is between four and seven, this method will usually produce a very precise experiment. In practice, the plots need not of course be actually square, but may be varied in shape to suit the requirements of cultivation. Precision is however sometimes lost if they are made too long and narrow.

The foregoing brief exposition is intended merely as a summary of the main principles involved in these methods of field experimentation. Those interested are referred to References (4) and (5) for a more detailed discussion of these points. Examples of the practical application of both Randomised Block and Latin Square arrangements, and also of the methods of computation most advantageously employed in dealing with the results, will be found in (6), (7), (10) and (18). Proofs of some of the mathematical theorems underlying the Analysis of Variance will be found in (11).

Interesting evidence in support of Fisher's views has been provided by Tedin (17) as a result of comparing the estimate of error obtained when randomly determined Latin Squares and also systematic arrangements in a square were superimposed upon the plot yields of uniformity trials reported in the literature.

4. Consideration of the Present Arrangement in the Light of these Principles

While the principles discussed in the foregoing section were put forward after the present experiment had been done, it may be instructive to consider in their light the field arrangement used. It is at once obvious that this does not fulfil the conditions set forth above. In the first place, the systematic alternation of the "check" and "rate" plots may have some feature in common with variations in soil fertility, thus rendering the comparisons subject to unknown errors: The same objection, moreover, applies to the grouping of all the plots of any one seed rate together. As a possible consequence of this

arrangement, we might get, say, the comparison " $2\frac{1}{2}$ bushels *versus* check" being made on land of a quite different level of fertility from that employed for the comparison " $3\frac{1}{2}$ bushels *versus* check". Then we have only to make the not altogether unwarranted assumption that the optimum rate of seeding is to some extent dependent on fertility, to see that our two sets of observations may not be truly comparable, and that if they are not, no amount of check plots will make them so.

For example, the comparison " $2\frac{1}{2}$ bushels *versus* 3 (check)" might be made on land such that the average yield of the three-bushel plots was 50 bushels per acre. Here, owing to the limited capacity of the soil to support a crop, and the consequent mutually detrimental competition of plants densely spaced, the plots sown at the lower rate might give the higher total yield. On the other hand, the comparison " $3\frac{1}{2}$ bushels *versus* 3" might be made on more fertile land, where the average yield of the three-bushel plots was, say, 75 bushels per acre. In this case, as a result of the more favorable conditions, the higher rate of seeding might well give a significantly higher return. The incautious might thus be led to assume that greater yields were to be obtained by seeding either above or below the three-bushel rate, a minimum return being obtained from seed rates in the neighborhood of three bushels. In fact, of course, the observed effect would be entirely spurious.

Extreme though this example may appear, we shall have cause to see, upon examination of the data, that situations of a similar nature involving differences in fertility level of quite the same order, did in fact occur.

Similar considerations apply to the segregation of all the plots of one variety upon one "range", or part, of the experimental block of land, with the added fact that here we have no means of determining to what extent soil differences between ranges may have entered as a disturbing factor.

A word must also be said concerning the rates of seeding adopted for the different varieties. It will be observed that these vary from 1 to 3 bushels in the case of Daubeney, from 2 to 4 in the case of Banner, and from $2\frac{1}{2}$ to $4\frac{1}{2}$ in the case of Abundance. The lowest rate at which Abundance was sown is thus only $\frac{1}{2}$ bushel less than the highest rate of Daubeney. This was done in order to centre the various rates employed for each variety upon the rate at which that variety was most commonly sown in agricultural practice. Since, however, one of the objects of the experiment was to ascertain whether varieties differing in seed and plant size and in tillering capacity should be sown at the same, or different, rates in variety trials, it would appear that a better arrangement would have been to apply a suitably-extended series of seed rates impartially to all three varieties.

It may be of some interest to illustrate the application of the principles of Section 3 by putting forward an alternative scheme to that here used. This may best be done by employing the doubly restricted random arrangement, or Latin Square.

The first point to be noted is that by discarding the check plot system the major proportion of the 99 check plots which it necessitated are eliminated. This makes it possible to extend the range of seed rates, whilst still keeping the

expenditure of land, labor, and materials considerably below that required by the check plot system. We may therefore decide to compare the behavior of all three varieties when sown at 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4 and $4\frac{1}{2}$ bushels per acre.

Six replicate plots of each of these seed rates are then arranged in a randomly determined Latin Square, as shown in the upper portion of Fig. 2. Each of the plots there represented by a square would actually consist of 27 drill rows $18\frac{1}{2}$ ft. long, spaced 7 in. apart; *i.e.*, three of our nine-row plots laid side by side. These plots are now each divided into three sub-plots of nine drill-rows, and to the three sub-plots in each rate-plot the three experimental varieties are allotted at random. The completed arrangement is shown in the lower portion of Fig. 2.

The restricted random arrangement allows soil differences between the rows and columns of large rate-plots to be eliminated, and at the same time makes possible a valid estimate of the remaining errors to which our comparisons are subject. The procedure of growing the three varieties in close proximity to each other in all the rate-plots allows attention to be concentrated on what is after all the main point at issue: namely, the differential response of these varieties to variations in the rate of seeding.

The centre row of each nine-row plot could be used for the determination of stand and tillering, although this method of sampling is by no means an ideal one. Discarding the two outside rows then leaves six rows from which to determine the yield. At harvest time one foot would of course be removed from each end of the rows, as before.

The total number of nine-row plots in this arrangement is 108. In the arrangement actually used it was 195. It thus appears that two Latin Squares, giving in all 12 plots of each kind, could be laid down with little more labor than was involved in the experiment as actually carried out. This should produce a rather precise experiment, the results of which could be accepted with some confidence. By reducing the size of the plots from nine to seven rows, which would mean the determination of yield from four rows, the labor requirements could be brought below those of the arrangement actually adopted. This would of course be achieved at the expense of a certain loss of accuracy.

Each Latin Square used should of course be a separately determined random arrangement.

5. Analysis of the Results

A summary of the observational data is given in Tables I, II, III and IV¹. This includes determinations of the weight per 1000 kernels and percentage germination of the seed used, the number of germinable seeds sown, and the mean values found at each seed rate for stand of plants, number of tillers and yield of grain. The mean values of observations made upon adjacent "check" and "rate" plots (see Fig. 1) appear on the same line.

Upon an examination of these, it is at once apparent that the differences in yield between the various check and rate plots are surprisingly small, being

¹The complete observational data are too voluminous to be practicable of publication, but are available for study in the Library of the National Research Council of Canada.

TABLE I
MEAN VALUES OF OBSERVATIONAL DATA, BY YEARS, VARIETIES AND RATES.
DOMINION EXPERIMENTAL STATION, CHARLOTTETOWN, P.E.I.

Year	Variety	Weight of 1000 kernels, gm.	Germination, %	Rate of seeding per acre (bushels)		Number of germinable seeds per acre (thousands)		Number of plants per acre (thousands)		Number of tillers per acre (thousands)		Stand of plants in % of germinable seeds sown		Number of tillers per plant		Yield of grain per acre (bushels)	
				Check	Rate	Check	Rate	Check	Rate	Check	Rate	Check	Rate	Check	Rate	Check	Rate
1924	Abundance	39.7	100	3	2½	1159	969	960	772	987	820	82.5	79.7	1.03	1.06	40.65	39.61
				3	3	1159	1353	982	1084	1023	1124	84.8	80.2	1.04	1.04	33.03	36.23
				3	4	1159	1548	953	1125	980	1148	82.2	72.7	1.03	1.02	31.95	32.83
1924	Banner	34.4	100	3	2	1159	1742	1000	1429	1064	1465	86.3	82.0	1.06	1.02	46.55	48.90
				3	3	1335	892	1183	782	1217	818	88.6	87.8	1.03	1.04	47.51	45.16
				3	4	1335	1113	1214	1019	1249	1059	90.9	91.7	1.03	1.04	53.33	51.70
1924	Daubency	21.2	100	3	2	1335	1557	1202	1375	1232	1402	90.0	88.3	1.03	1.02	50.43	50.57
				3	3	1335	1783	1161	1556	1204	1585	87.0	87.3	1.04	1.02	64.53	65.69
				2	1	1448	724	1203	621	1500	956	83.3	85.7	1.24	1.54	33.08	28.08
1925	Abundance	32.2	100	2	1½	1448	1086	1206	955	1574	1419	83.2	88.0	1.30	1.40	30.96	28.39
				2	2½	1448	1810	1252	1563	1543	1839	86.4	86.3	1.23	1.18	33.06	34.53
				2	3	1448	2172	1263	1783	1883	2119	87.2	82.1	1.49	1.20	40.84	47.34
1925	Banner	30.1	100	3	2½	1431	1193	1203	971	1402	1226	83.7	81.4	1.16	1.25	35.91	36.03
				3	3	1431	1670	1163	1331	1306	1552	81.3	79.7	1.13	1.15	44.58	48.07
				3	4	1431	1909	1166	1553	1340	1760	82.4	80.4	1.17	1.13	52.54	47.66
1925	Daubency	20.7	100	2	1	1482	741	1019	522	1339	819	68.7	69.6	1.32	1.56	33.20	23.94
				2	1½	1482	1111	1203	1138	1432	1271	65.6	68.9	1.29	1.32	42.13	24.44
				2	3	1482	1852	993	1334	1695	2050	66.9	66.0	1.80	1.58	48.13	39.01
1926	Abundance	33.6	95	3	2½	1303	1086	1087	817	1103	832	82.7	75.2	1.02	1.63	26.04	25.32
				3	3	1303	1521	1050	1252	1073	1275	80.5	82.3	1.02	1.02	39.18	39.05
				3	4	1303	1738	1067	1344	1095	1363	81.8	77.3	1.03	1.02	43.91	45.08
1926	Banner	33.6	99	3	2	1352	910	1004	674	1018	700	78.3	74.0	1.08	1.04	44.55	42.39
				3	2½	1352	1136	1092	812	1142	812	76.1	76.1	1.08	1.01	46.50	46.50
				3	4	1352	1559	985	1209	945	1226	72.3	72.0	1.03	1.02	56.90	56.53
1926	Daubency	25.2	96	2	1	1177	588	856	363	1594	941	72.7	61.6	2.61	2.61	64.95	56.98
				2	1½	1177	883	853	645	1382	1306	72.4	73.1	1.72	1.83	51.57	49.24
				2	2½	1177	1466	886	1014	1506	1562	75.3	69.2	1.71	1.60	57.57	56.91
1926	Daubency	25.2	96	2	3	1177	1761	926	1247	1719	1826	78.7	70.9	1.77	1.46	61.54	61.54

TABLE II
MEAN VALUES OF OBSERVATIONAL DATA, BY YEARS, VARIETIES AND RATES.
MACDONALD COLLEGE, STE. ANNE DE BELLEVUE, QUEBEC

Year	Variety	Weight of 100 seeds, gm.	Germination, %	Rate of seeding per acre (bushels)		Number of plants per acre (thousands)		Number of tillers per acre (thousands)		Stand of plants in % of germinable seeds sown		Number of tillers per plant		Yield of grain per acre (bushels)	
				Check	Rate	Check	Rate	Check	Rate	Check	Rate	Check	Rate	Check	Rate
1924	Abundance	39.0	99	3	2 1/2	1175	979	617	541	785	723	52.5	55.2	52.34	54.34
				3	3 1/2	1175	1371	770	990	914	1129	65.5	72.2	43.71	43.86
				3	4 1/2	1175	1567	579	803	620	992	42.3	51.3	41.62	43.83
	Banner	34.2	99	3	2	1175	1762	523	834	620	992	39.3	47.3	52.02	54.04
				3	2 1/2	1340	893	623	357	646	478	39.1	39.9	49.52	40.64
				3	3 1/2	1340	1116	912	696	1017	788	68.1	62.3	54.98	54.34
	Daubeny	22.8	98	3	3 1/2	1340	1563	811	877	903	997	60.5	56.1	52.58	55.72
				3	4	1340	1786	602	823	731	1001	44.9	46.1	51.81	52.49
				2	1	1326	663	480	312	795	604	36.2	47.1	42.27	41.80
	Banner	29.3	98	2	1 1/2	1326	1033	718	986	978	1212	54.1	55.0	37.42	38.04
				2	2 1/2	1326	1658	596	1108	876	1404	44.9	55.7	37.46	38.04
				2	3	1326	1990	596	1108	876	1404	44.9	55.7	50.93	46.65
1925	Daubeny	23.0	96	3	2	1549	1033	1068	847	1241	958	68.9	82.0	60.90	57.28
				3	2 1/2	1549	1291	993	862	1302	1154	63.0	66.8	72.97	69.73
				3	3 1/2	1549	1807	871	974	1229	1299	56.3	53.9	70.86	69.81
	Daubeny	23.0	96	3	4	1549	2065	859	935	1182	1469	55.5	45.3	72.51	73.21
				2	1	1288	644	923	564	1471	1059	71.7	87.6	58.78	50.26
				2	1 1/2	1288	966	915	772	1447	1381	70.4	78.1	62.91	62.19
	Abundance	33.6	95	2	2 1/2	1288	1610	999	1094	1646	1689	77.6	68.1	68.00	68.51
				2	3	1288	1932	1026	1078	1734	1966	79.7	55.8	64.82	65.02
				3	2 1/2	1362	1135	755	730	1054	1006	55.5	64.3	58.68	59.47
	Banner	33.6	99	3	3 1/2	1362	1589	755	943	1056	1241	55.4	59.4	59.19	62.68
				3	4	1362	1816	786	1022	1146	1454	57.8	56.3	66.53	65.74
				3	4 1/2	1362	2043	823	1033	1138	1557	60.5	50.6	64.53	63.09
	Daubeny	25.2	96	3	2	1366	910	723	501	1140	890	53.1	55.1	72.85	71.76
				3	2 1/2	1366	1138	1039	706	1166	1028	76.1	62.0	74.08	75.77
				3	3 1/2	1366	1593	1006	787	1093	1252	73.7	49.4	74.69	73.88
	Daubeny	25.2	96	3	4	1366	1821	786	782	1089	1304	57.6	43.0	71.61	72.84
				2	1	1223	589	787	456	1075	865	64.4	77.5	47.52	41.84
				2	1 1/2	1223	820	676	537	1240	1343	55.3	60.2	47.53	47.00
				2	2 1/2	1223	1031	676	537	1240	1343	55.3	60.2	64.27	62.00
				2	3	1223	1840	633	838	1272	1431	51.8	45.6	64.49	66.68

TABLE III
MEAN VALUES OF OBSERVATIONAL DATA, BY YEARS, VARIETIES AND RATES.
DOMINION EXPERIMENTAL STATION, SCOTT, SASKATCHEWAN

Year	Variety	Weight of 1000 kernels, gm.	Germination, %	Rate of seeding per acre (bushels)		Number of germinable seeds per acre (thousands)		Number of plants per acre (thousands)		Number of tillers per acre (thousands)		Stand of plants in % of germinable seeds sown		Number of tillers per plant		Yield of grain per acre (bushels)	
				Check	Rate	Check	Rate	Check	Rate	Check	Rate	Check	Rate	Check	Rate	Check	Rate
1924	Abundance	39.0	96	3	2 1/2	1140	946	978	858	997	870	85.3	89.8	1.02	1.01	25.77	27.14
				3	3 1/4	1140	1334	987	1220	1000	1226	86.5	91.6	1.01	1.01	28.47	27.14
				3	4	1140	1516	1022	1335	1044	1295	89.5	88.1	1.01	1.00	24.92	22.17
1924	Banner	34.2	100	3	2	1353	901	1153	763	1192	868	91.7	84.5	1.03	1.12	37.11	36.17
				3	2 1/2	1353	1177	1170	1022	1125	1125	82.8	83.7	1.01	1.03	32.66	34.54
				3	4	1353	1801	1139	1502	1148	1506	84.1	83.2	1.01	1.00	25.07	22.12
1924	Daubeney	22.8	95	2	1	1285	643	1144	556	1190	775	88.8	86.6	1.04	1.39	32.38	29.79
				2	1 1/2	1285	964	1114	873	1136	914	86.5	90.4	1.02	1.05	15.10	15.86
				2	2	1285	1607	1148	1351	1161	1366	89.1	84.1	1.01	1.01	15.86	13.76
1925	Abundance	36.4	98	3	2 1/2	1247	1039	1130	945	1151	1071	89.6	91.0	1.03	1.03	82.84	80.57
				3	3 1/4	1247	1554	1150	1350	1211	1384	92.2	86.0	1.12	1.14	87.28	90.66
				3	4	1247	1662	1160	1552	1224	1557	94.6	93.4	1.04	1.01	83.25	84.61
1925	Banner	30.8	98	3	2	1471	981	1359	938	1404	1069	92.4	95.6	1.05	1.14	88.10	86.16
				3	2 1/2	1471	1226	1367	1156	1333	1173	92.4	94.3	1.08	1.05	88.97	95.05
				3	3 1/4	1471	1716	1376	1628	1350	1565	93.6	95.0	1.00	1.00	86.90	84.83
1925	Daubeney	21.9	96	2	1	1355	677	1267	622	1393	1458	93.6	91.8	1.15	1.34	72.04	72.08
				2	1 1/2	1355	1016	1277	932	1575	1477	94.3	91.7	1.23	1.52	72.64	69.64
				2	2	1355	1693	1255	1547	1619	1502	92.7	91.3	1.28	1.06	72.30	69.70
1926	Abundance	33.6	95	3	2 1/2	1306	1088	982	837	1187	960	75.2	76.9	1.21	1.15	38.75	40.09
				3	3 1/4	1306	1524	942	1121	1167	1318	72.1	73.6	1.25	1.18	36.96	37.82
				3	4	1306	1742	951	1314	1168	1536	72.1	73.3	1.25	1.17	33.48	35.48
1926	Banner	33.6	99	3	2	1365	910	974	691	1194	852	71.4	76.0	1.23	1.23	54.60	54.79
				3	2 1/2	1365	1138	924	809	1120	1022	67.7	71.1	1.21	1.27	56.68	57.89
				3	3 1/4	1365	1593	913	1008	1099	1178	66.9	63.3	1.21	1.17	47.24	46.70
1926	Daubeney	25.2	96	2	1	1177	589	809	383	1010	800	68.7	65.1	1.25	2.09	35.85	34.89
				2	1 1/2	1177	883	766	625	940	863	55.1	50.4	1.43	1.38	34.33	34.52
				2	2 1/2	1177	1422	771	1036	1111	1316	64.7	70.4	1.31	1.11	30.33	30.33

TABLE IV
MEAN VALUES OF OBSERVATIONAL DATA, BY YEARS, VARIETIES AND RATES,
UNIVERSITY OF ALBERTA, EDMONTON, ALBERTA

Year	Variety	Weight of 1000 kernels, gm.	Germination, %	Rate of seeding per acre (bushels)		Number of germinable seeds per acre (thousands)		Number of plants per acre (thousands)		Number of tillers per acre (thousands)		Stand of plants in % of germinable seeds sown		Number of tillers per plant		Yield of grain per acre (bushels)	
				Check	Rate	Check	Rate	Check	Rate	Check	Rate	Check	Rate	Check	Rate	Check	Rate
1924	Abundance	39.0	96	3	2½	1131	941	982	837	1155	1060	86.8	88.9	1.18	1.30	92.10	84.92
				3	3	1131	1322	983	1092	1095	1232	86.9	82.6	1.11	1.13	84.82	80.82
				3	4	1131	1512	977	1303	1160	1431	86.4	86.2	1.19	1.09	90.03	94.12
				3	4½	1131	1697	1028	1471	1295	1563	90.9	86.7	1.26	1.06	112.04	110.80
	Banner	34.2	100	3	2	1344	896	1280	1113	1308	1150	95.3	124.2	1.02	1.04	122.61	129.45
				3	2½	1344	1344	1202	1302	1163	1211	88.4	103.6	1.01	1.02	124.71	126.89
				3	3	1344	1576	1299	1388	1307	1393	96.7	88.4	1.01	1.01	133.12	129.25
				3	4	1344	1792	1180	1384	1185	1385	87.8	77.2	1.00	1.00	124.35	120.70
	Daubney	22.8	95	2	1	1276	638	1135	657	2535	2425	89.0	102.9	2.26	3.74	87.71	81.44
				2	1½	1276	959	1171	872	2389	2459	91.8	90.9	2.08	2.79	84.56	81.35
				2	2	1276	1598	1240	1311	2634	2630	97.1	82.0	2.14	2.02	78.82	79.80
				2	3	1276	1919	1145	1182	2556	2292	89.7	61.6	2.07	1.89	86.52	86.46
1925	Abundance	28.7	99	3	2½	1589	1322	1285	1066	1313	1111	80.9	80.7	1.02	1.04	94.22	99.67
				3	3	1589	1851	1305	1513	1310	1517	82.1	81.7	1.00	1.00	89.32	91.73
				3	4	1589	2118	1289	1724	1305	1752	81.1	81.4	1.01	1.00	91.58	77.46
				3	4½	1589	2381	1338	1942	1364	1960	84.2	81.6	1.02	1.01	99.36	95.51
	Banner	29.1	98	3	2	1548	1032	1324	875	1340	1018	85.5	84.8	1.01	1.16	123.55	120.60
				3	2½	1548	1290	1315	1095	1348	1159	85.0	84.9	1.03	1.06	126.60	127.02
				3	3	1548	1806	1350	1547	1356	1551	87.2	85.1	1.03	1.03	116.72	117.02
				3	4	1548	2068	1328	1761	1360	1763	85.4	83.1	1.01	1.00	114.15	110.92
	Daubney	21.1	96	2	1	1394	697	1102	574	1590	1358	79.1	82.4	1.44	2.36	101.67	94.37
				2	1½	1394	1045	1193	911	1646	1556	85.6	87.2	1.38	1.70	106.15	101.73
				2	2	1394	1747	1215	1547	1622	1795	87.1	88.5	1.34	1.16	99.97	102.91
				2	3	1394	2095	1107	1607	1576	1830	79.4	76.7	1.42	1.14	105.18	106.77
1926	Abundance	33.5	95	3	2½	1303	1066	1112	1027	1116	1037	85.3	94.6	1.00	1.01	89.02	88.56
				3	3	1303	1303	1121	1185	1188	1213	89.7	86.6	1.01	1.09	78.14	75.58
				3	4	1303	1722	1189	1509	1386	1513	90.7	86.6	1.01	1.00	89.17	89.06
				3	4½	1303	1955	1126	1540	1140	1547	86.4	78.8	1.01	1.00	89.51	88.14
	Banner	35.5	99	3	2	1285	855	1062	748	1087	795	84.2	87.4	1.00	1.06	84.38	82.46
				3	2½	1285	1068	1133	922	1153	959	88.1	86.3	1.02	1.04	91.30	95.17
				3	3	1285	1498	1177	1230	1182	1247	91.6	82.1	1.00	1.02	88.48	87.59
				3	4	1285	1711	1156	1352	1170	1354	90.0	79.0	1.01	1.00	102.68	99.11
	Daubney	25.5	96	2	1	1154	570	991	530	1427	1310	85.3	91.4	1.45	2.48	68.67	65.43
				2	1½	1154	844	903	684	1501	1338	78.1	79.1	1.66	1.95	73.88	72.34
				2	2	1154	1444	899	970	1164	1164	77.0	67.2	1.44	1.20	67.64	68.45
				2	3	1154	1733	924	1358	1245	1488	80.0	78.3	1.35	1.10	69.12	73.31

often only of the order of a bushel or so per acre. The first step in the analysis would therefore appear to be to ascertain whether in fact the differences in seed rate do appear to have influenced yield to any significant extent, or whether such variations as have occurred might reasonably be regarded as purely chance effects. This may be done in the following manner: from the sum of the yields of the two three-row plots constituting each large rate-plot we may subtract the yields of the two three-row check plots immediately adjacent thereto (see Fig. 1). In this way, a set of eight differences is obtained for each comparison. To test if the mean of these eight differences departs significantly from zero we calculate the quantity

$$t = \frac{\bar{x} \sqrt{n^1(n^1-1)}}{\sqrt{S(x-\bar{x})^2}}$$

Where \bar{x} is the mean of the eight observed differences, S indicates summation over the eight individuals, and $n^1=8$. The actual process of calculation is simplified by noting that

$$\frac{\bar{x} \sqrt{n^1(n^1-1)}}{\sqrt{S(x-\bar{x})^2}} = \frac{S(x)}{\sqrt{S(x-\bar{x})^2}} \cdot \frac{\sqrt{n^1-1}}{\sqrt{n^1}}$$

This procedure was applied to the whole of the observed grain yields, with the results shown in Table V. Owing to shortage of seed, gnawing of sheaves by mice, and various other causes, the number of plots from which yields were available was in some cases reduced. The second column of the table shows the number of differences it was possible to form in these cases. The fourth column gives the sum of the squares of the deviations of the observed differences from their mean, and the fifth column the values of the ratio t .

Now the distribution of the statistic t is known in the case when our two sets of observations are random samples from a homogeneous normal population, and the probability that any value of t obtained should lie between certain fixed limits may be found from a table of "Student's" integral (5, p.139). In making tests of significance upon isolated samples, therefore, it is customary to ascertain from the table what is the probability of obtaining by chance a value of t equal to or greater than that actually observed. If this probability is sufficiently small, we then say with some confidence that the observed effects are not of a purely random nature, but that the various treatments, etc., employed have significantly influenced the results. Values having a probability of 0.05 or less are usually regarded as significant in agricultural experimentation. Such values have been marked with an asterisk (*) in Table V.

In the present case, however, we may carry our test of significance one stage further. The total number of values of t in Table V, based on seven degrees of freedom ($n^1=8$) is 113. From the table of t we may determine the proportion of these expected to fall between different limits. Then, with this "expected" distribution we may compare the distribution actually obtained. This has been done in Table VI.

TABLE V

RATIO OF OBSERVED DIFFERENCES IN YIELD TO THEIR ESTIMATED STANDARD ERROR

Rate of seeding comparison			n^1	$S(x)$	$S(x-\bar{x})^2$	t
Charlottetown						
1924	Abundance	2½ bu. v. 3 bu.	8	-16.70	294.70	0.915
		3½ bu. v. 3 bu.	8	51.08	291.67	2.798*
		4 bu. v. 3 bu.	8	14.08	98.75	1.325
		4½ bu. v. 3 bu.	7	30.27	215.46	1.909
	Banner	2 bu. v. 3 bu.	8	-37.59	648.32	1.381
		2½ bu. v. 3 bu.	8	-26.14	446.94	1.157
		3½ bu. v. 3 bu.	8	2.18	255.91	0.128
		4 bu. v. 3 bu.	7	10.39	729.64	0.356
	Daubeney	1 bu. v. 2 bu.	8	-79.97	210.70	5.142*
		1½ bu. v. 2 bu.	8	-41.16	74.49	4.460*
		2½ bu. v. 2 bu.	8	23.66	190.06	1.606
		3 bu. v. 2 bu.	7	96.84	312.88	5.068*
1925	Abundance	2½ bu. v. 3 bu.	7	13.83	382.32	0.655
		3½ bu. v. 3 bu.	7	57.38	1010.43	1.671
		4 bu. v. 3 bu.	8	34.06	1055.02	0.981
		4½ bu. v. 3 bu.	8	37.93	880.02	1.196
	Banner	2 bu. v. 3 bu.	8	-189.54	1166.66	5.191*
		2½ bu. v. 3 bu.	8	-34.27	667.84	1.240
		3½ bu. v. 3 bu.	8	33.63	3567.85	0.527
		4 bu. v. 3 bu.	8	30.98	1267.17	1.814
	Daubeney	1 bu. v. 2 bu.	8	-148.15	533.21	6.001*
		1½ bu. v. 2 bu.	8	-132.69	700.19	4.691*
		2½ bu. v. 2 bu.	8	70.16	1110.17	1.970
		3 bu. v. 2 bu.	8	36.09	380.13	1.732
1926	Abundance	2½ bu. v. 3 bu.	8	-11.41	115.56	0.993
		3½ bu. v. 3 bu.	8	-2.04	107.02	0.184
		4 bu. v. 3 bu.	8	18.73	122.16	1.585
		4½ bu. v. 3 bu.	8	57.09	269.82	3.251*
	Banner	2 bu. v. 3 bu.	8	-34.69	210.00	2.239
		2½ bu. v. 3 bu.	8	-10.89	1251.21	0.288
		3½ bu. v. 3 bu.	8	-5.93	386.15	0.282
		4 bu. v. 3 bu.	8	2.69	102.76	0.248
	Daubeney	1 bu. v. 2 bu.	8	-127.44	2201.05	2.541*
		1½ bu. v. 2 bu.	8	-37.15	702.33	1.311
		2½ bu. v. 2 bu.	8	-10.51	840.81	0.339
		3 bu. v. 2 bu.	8	48.94	689.54	1.743
Macdonald College						
1924	Abundance	2½ bu. v. 3 bu.	8	31.95	366.48	1.561
		3½ bu. v. 3 bu.	8	2.35	64.03	0.274
		4 bu. v. 3 bu.	8	34.95	549.47	1.394
		4½ bu. v. 3 bu.	8	32.40	684.89	1.158
	Banner	2 bu. v. 3 bu.	8	1.87	677.81	0.067
		2½ bu. v. 3 bu.	8	-10.31	282.99	0.573
		3½ bu. v. 3 bu.	8	50.22	838.96	1.634
		4 bu. v. 3 bu.	8	10.78	213.37	0.690
	Daubeney	1 bu. v. 2 bu.	8	-7.42	494.23	0.312
		1½ bu. v. 2 bu.	8	13.72	123.70	1.154
		2½ bu. v. 2 bu.	8	25.35	301.13	1.366
		3 bu. v. 2 bu.	8	-68.48	272.01	3.884

TABLE V—Continued

Rate of seeding comparison			n^1	$S(x)$	$S(x-\bar{x})^2$	t
1925	Banner	2 bu. v. 3 bu.	8	-57.90	923.76	1.782
		2½ bu. v. 3 bu.	8	-51.84	1021.07	1.518
		3½ bu. v. 3 bu.	8	-16.84	329.45	0.868
		4 bu. v. 3 bu.	8	11.23	380.08	0.520
	Daubeney	1 bu. v. 2 bu.	8	-136.40	527.49	5.555*
		1½ bu. v. 2 bu.	8	-11.56	259.26	0.672
		2½ bu. v. 2 bu.	8	8.28	323.81	0.431
		3 bu. v. 2 bu.	8	3.26	479.46	0.139
1926	Abundance	2½ bu. v. 3 bu.	5	14.14	660.23	0.492
		3½ bu. v. 3 bu.	8	55.82	326.94	2.888*
		4 bu. v. 3 bu.	8	-12.65	1418.28	0.314
		4½ bu. v. 3 bu.	8	8.96	207.22	0.586
	Banner	2 bu. v. 3 bu.	3	-6.49	0.91	5.544*
		2½ bu. v. 3 bu.	8	27.00	569.26	1.058
		3½ bu. v. 3 bu.	8	-12.97	252.92	0.763
		4 bu. v. 3 bu.	8	19.76	333.21	1.012
	Daubeney	1 bu. v. 2 bu.	8	-90.84	120.60	7.738*
		1½ bu. v. 2 bu.	8	-37.38	370.58	1.816
		2½ bu. v. 2 bu.	8	27.43	120.64	2.336
		3 bu. v. 2 bu.	8	35.03	490.34	1.480
Scott, 1924	Abundance	2½ bu. v. 3 bu.	8	21.82	120.51	1.859
		3½ bu. v. 3 bu.	8	-21.43	234.06	1.310
		4 bu. v. 3 bu.	7	-38.34	431.52	1.709
		4½ bu. v. 3 bu.	6	-28.23	95.05	2.643*
	Banner	2 bu. v. 3 bu.	8	-14.95	302.38	0.804
		2½ bu. v. 3 bu.	8	29.92	302.25	1.610
		3½ bu. v. 3 bu.	7	-41.41	409.38	1.895
		4 bu. v. 3 bu.	7	-10.41	167.61	0.744
	Daubeney	1 bu. v. 2 bu.	8	-41.53	554.34	1.469
		1½ bu. v. 2 bu.	8	12.12	38.65	1.824
		2½ bu. v. 2 bu.	8	-33.30	191.22	2.252
		3 bu. v. 2 bu.	7	-13.23	103.90	1.202
1925	Abundance	2½ bu. v. 3 bu.	7	-8.88	1006.68	0.259
		3½ bu. v. 3 bu.	8	54.08	1059.66	1.554
		4 bu. v. 3 bu.	8	21.83	1017.87	0.636
		4½ bu. v. 3 bu.	7	-27.66	1514.38	0.658
	Banner	2 bu. v. 3 bu.	8	-30.91	617.54	1.164
		2½ bu. v. 3 bu.	8	97.22	1835.14	2.123
		3½ bu. v. 3 bu.	8	-33.14	1061.52	0.951
		4 bu. v. 3 bu.	7	-56.17	1600.57	1.300
	Daubeney	1 bu. v. 2 bu.	8	0.71	887.37	0.030
		1½ bu. v. 2 bu.	8	-48.17	708.75	1.692
		2½ bu. v. 2 bu.	8	-41.59	878.99	1.312
		3 bu. v. 2 bu.	7	-5.61	565.81	0.218
1926	Abundance	2½ bu. v. 3 bu.	8	21.21	262.39	1.225
		3½ bu. v. 3 bu.	8	13.53	120.11	1.155
		4 bu. v. 3 bu.	8	-18.82	146.81	1.453
		4½ bu. v. 3 bu.	7	-86.77	313.28	4.539*

TABLE V—Continued

Rate of seeding comparison			n^1	$S(x)$	$S(x-\bar{x})^2$	t	
1926	Banner	2 bu. v. 3 bu.	8	3.15	533.93	0.128	
		2½ bu. v. 3 bu.	8	19.44	472.81	0.836	
		3½ bu. v. 3 bu.	8	-8.64	839.56	0.279	
		4 bu. v. 3 bu.	7	-22.94	54.89	2.867*	
	Daubeney	1 bu. v. 2 bu.	8	-15.33	298.23	0.830	
		1½ bu. v. 2 bu.	8	-16.42	284.29	0.911	
		2½ bu. v. 2 bu.	8	19.64	1451.26	0.482	
		3 bu. v. 2 bu.	7	18.54	266.06	1.063	
University of Alberta							
1924	Abundance	2½ bu. v. 3 bu.	8	-114.14	1791.69	2.522*	
		3½ bu. v. 3 bu.	8	-63.98	6303.02	0.754	
		4 bu. v. 3 bu.	4	32.71	182.61	2.096	
		4½ bu. v. 3 bu.	4	-9.92	943.39	0.280	
	Banner	2 bu. v. 3 bu.	6	101.67	1291.18	2.583*	
		2½ bu. v. 3 bu.	8	34.81	2058.27	0.717	
		3½ bu. v. 3 bu.	6	-42.86	3622.57	2.056	
		4 bu. v. 3 bu.	6	-40.75	834.21	1.288	
	Daubeney	1 bu. v. 2 bu.	8	-100.31	164.56	7.315*	
		1½ bu. v. 2 bu.	8	-51.43	953.66	1.558	
		2½ bu. v. 2 bu.	8	17.32	1659.85	0.398	
		3 bu. v. 2 bu.	8	-3.96	3031.06	0.067	
	1925	Abundance	2½ bu. v. 3 bu.	8	87.28	1774.69	1.938
			3½ bu. v. 3 bu.	8	38.59	489.89	1.631
			4 bu. v. 3 bu.	8	-225.90	1626.49	5.240*
			4½ bu. v. 3 bu.	7	-51.44	2072.59	1.046
		Banner	2 bu. v. 3 bu.	8	-47.10	811.77	1.546
			2½ bu. v. 3 bu.	8	6.59	1236.38	0.175
			3½ bu. v. 3 bu.	8	35.49	467.00	1.536
			4 bu. v. 3 bu.	8	-51.70	55.65	6.483*
Daubeney		1 bu. v. 2 bu.	8	-116.80	1087.01	3.314*	
		1½ bu. v. 2 bu.	8	-70.77	355.61	3.510*	
		2½ bu. v. 2 bu.	8	46.91	909.92	1.455	
		3 bu. v. 2 bu.	8	22.65	310.30	1.190	
1926	Abundance	2½ bu. v. 3 bu.	8	-7.32	805.16	0.241	
		3½ bu. v. 3 bu.	8	-40.92	1311.30	1.057	
		4 bu. v. 3 bu.	8	-1.74	651.26	0.638	
		4½ bu. v. 3 bu.	8	-21.88	3624.45	0.340	
	Banner	2 bu. v. 3 bu.	8	-30.72	667.40	1.112	
		2½ bu. v. 3 bu.	8	61.98	1442.20	1.527	
		3½ bu. v. 3 bu.	8	-14.23	588.08	0.549	
		4 bu. v. 3 bu.	7	-51.47	570.32	1.995	
	Daubeney	1 bu. v. 2 bu.	8	-51.91	403.56	2.417*	
		1½ bu. v. 2 bu.	8	-24.62	556.42	0.976	
		2½ bu. v. 2 bu.	8	12.95	269.39	0.738	
		3 bu. v. 2 bu.	8	66.92	649.79	2.456*	

TABLE VI
DISTRIBUTION OF VALUES OF t

t	"Ex- pected"	Actual
0.000		
0.130	11.3	5
0.263	11.3	5
0.402	11.3	9
0.549	11.3	5
0.711	11.3	6
0.896	11.3	9
1.119	11.3	10
1.415	11.3	15
1.895	11.3	23
2.365	5.65	6
2.998	3.77	6
3.499	1.13	2
∞	1.13	12

The distribution of the values of t actually obtained is clearly not in accord with that expected as a result of purely random fluctuations in the yields of the various plots. There is a deficiency of the lower values and a notable excess of the higher ones. We therefore conclude that, taking the experiment as a whole, the plots sown at different rates have shown a significant tendency to differ in yield.

The individual "significant differences" in Table V are not very numerous, and are scattered throughout the table. It appeared that the method of *regression* might be used to advantage in examining more closely into the nature and extent of the associated variations in seed rate and yield. Before proceeding with this, however, it may be of some interest to consider briefly the quantities $S(x - \bar{x})^2$ of Table V. These furnish a direct measure of the degree of *precision* attained in the experiment at the various stations in different years.

By addition, a total sum of squares of deviations of observed differences from their respective means is obtained for each station in each year. On division by the appropriate number of degrees of freedom these give the mean squares of Table VII. Obviously, the nearer the eight differences belonging to each

comparison approach to equality, the smaller will be their mean square deviation and the more trustworthy will be our results. It may be urged, however, that the actual values of the mean square deviation, or *variance*, of Table VII are not directly comparable. Thus it is true that the variance of the results

TABLE VII
ACTUAL AND PERCENTAGE VARIANCE OF RESULTS

Station	1924	1925	1926	Mean
Charlottetown	46.54 260.22	155.13 949.95	83.31 341.65	91.40 517.27
Macdonald College	57.96 257.66	75.79 176.76	64.09 144.93	64.74 193.12
Scott	37.03 662.60	160.68 240.02	62.27 383.79	86.97 428.80
University of Alberta	326.33 318.99	209.28 191.22	139.03 204.86	219.30 238.02
Mean	109.79 374.87	156.60 389.49	87.82 268.81	117.18 344.39

obtained at the University of Alberta in 1924 is 326.33, whilst the corresponding figure for Scott is only 37.03. But it is also true that the mean yield of all plots at the University of Alberta in 1924 was 101.14 bushels per acre, whereas at Scott it was only 23.64 bushels per acre. The percentage variance is therefore shown below the actual variance in each case. This was obtained by multiplying the actual variance by $\left(\frac{100}{m}\right)^2$, where m is the mean yield of all plots for the particular station and season under consideration.

The individual percentage variance values exhibit considerable fluctuation. Considering the experiment as a whole, however, we see that there is little difference in the level of accuracy attained in the first two years; the mean variance in the third year being somewhat lower. The different stations fall into two groups. Scott and Charlottetown show a mean variance, over the three years of the experiment, of 428.80 and 517.27 respectively, whilst the corresponding figures for the University of Alberta and Macdonald College are 193.12 and 238.02.

The point, as to whether there exist any significant differences in precision, may be tested by performing an Analysis of Variance (5, p.190 *et seq.*) upon the 12 individual values. The sum of the squares of the deviations of the 12 values from their general mean is divided into three portions, as in Table VIII. The mean squares are obtained from the sums of squares by dividing by the appropriate number of degrees of freedom. Considering now these mean squares, it is apparent that the largest is that ascribable to differences in the average variance (over the three-year period) at the four stations. Fluctuation of the average percentage variance in the different years gives rise to the smallest mean square, interaction of place and year occupying an intermediate position. As an illustration of this interaction, in 1924 the percentage variance at Scott and Charlottetown respectively was 662.60 and 260.22. In 1925, however, the corresponding figures were 240.02 and 949.95.

TABLE V:II
ANALYSIS OF VARIANCE OF PERCENTAGE VARIANCE

	Degrees of freedom	Sum of squares	Mean square
Between years	2	34701.51	17350.75
Between places	3	213164.24	71054.74
Interaction of years and places	6	358603.02	59767.20
Total	11	606468.77	

The test, whether the mean square ascribable to one of these causes differs significantly from that ascribable to any other, is performed by calculating the quantity Z , equal to half the difference of the natural logarithms of the two mean squares, and consulting the table of Z (5, p.212) to discover if the observed value might reasonably be expected as a result of chance. In the present case, none of the values of Z obtainable are significant. We therefore conclude that, although the differences in percentage variance of yield between stations are considerable, yet taken as a whole the accuracy of the results obtained, both as between stations and between years, is sensibly of the same order.

TABLE IX
MEAN YIELD OF ALL PLOTS, IN BUSHELS PER ACRE

	1924	1925	1926	Mean
Charlottetown	42.29	40.41	49.38	44.03
Macdonald College	47.43	65.48	66.50	59.80
Scott	23.64	81.82	40.28	48.58
University of Alberta	101.14	104.61	82.38	96.04
Mean	53.62	73.08	59.64	62.11

The mean yield of all plots at the various stations during the three years, used in the calculation of the percentage variances, may be of some interest. The actual values are given in Table IX.

TABLE X
ANALYSIS OF VARIANCE OF MEAN YIELD

	Degrees of freedom	Sum of squares	Mean square
Between years	2	796.45	398.28
Between places	3	5001.94	1667.31
Interaction of years and places	6	1561.44	260.24
Total	11	7359.83	

The variance of these mean yields may be analyzed in precisely the manner adopted in the case of the percentage variance. Table X shows the results of this procedure.

The mean square attributable to differences between places is definitely larger than the interaction mean square. The value

of Z is 0.9287, whereas the probability of obtaining by chance a value equal to or greater than 0.7798 is only 0.05. It is therefore legitimate to conclude that there are significant differences in the general level of fertility at the four stations. The nature of these differences is apparent from Table IX. On the other hand, the differences in yield from year to year of the four stations taken as a whole are not significantly greater than those attributable to the interaction of place and year.

In endeavoring to estimate the regression of yield on seed rate, the field arrangement precludes our using the actual yields of plots sown at different rates, since these are obviously not directly comparable. We must instead utilize the mean differences in yield between the 16 plots of each seed rate and the adjacent 16 check plots grown at each station each year. In each case there will be four such differences, which may be regarded as values of a dependent variate. The differences in seed rate between the check and rate plots provide the corresponding values of the independent variate. Denoting the latter by x and the former by y , we calculate as our estimate of the coefficient of regression of y on x

$$b = \frac{S(x-\bar{x})(y-\bar{y})}{S(x-\bar{x})^2}$$

where \bar{x} and \bar{y} are the mean values of the two quantities respectively and S

indicates summation over all four individuals. The values of b thus obtained for the different varieties, places and years are set forth in Table XI.

To test whether any particular value of b differs significantly from zero, we calculate the quantities

$$s^2 = \frac{1}{n^2 - 2} S(y - Y)^2 \quad (n^2 = 4)$$

where y is the observed value and Y the corresponding value calculated from the regression equation, and

$$t = \frac{b\sqrt{S(x - \bar{x})^2}}{s}$$

The probability of obtaining by chance any observed value of t is then found from the table of "Student's" distribution. When this probability is 0.05 or less, the corresponding values of b are regarded as significant, and are marked with an asterisk in Table XI.

TABLE XI
AVERAGE INCREASE IN YIELD (BUSHELS PER ACRE) FOR EACH HALF-BUSHEL
INCREASE IN RATE OF SEEDING

	Abundance	Banner	Daubeney
Charlottetown 1924	0.685	0.795*	2.739*
1925	0.286	3.181	3.571
1926	1.001	0.498	2.371
Macdonald College 1924	0.075	0.490	-0.691*
1925	...	1.083*	1.870
1926	-0.419	0.214	1.979*
Scott 1924	-1.163*	-0.444	0.046
1925	-0.211	-1.231	-0.048
1926	-1.706	-0.542	0.682*
University of Alberta 1924	2.051	-2.946	1.634
1925	-3.417	0.123	2.519
1926	-0.092	0.643	1.720*

A study of Table XI indicates that out of the 35 regression coefficients obtained, only 8 may be regarded as definitely significant. In all the other cases the variations in yield observed were either so small, or else so irregular in nature that the resulting values of b could quite reasonably be supposed to have arisen purely through chance. Nevertheless, an examination of the figures does allow us to deduce certain conclusions which are not altogether devoid of interest.

It will be observed that of the eight significant coefficients, five are associated with the variety Daubeney, only two with Banner, and only one with Abundance. Furthermore, in the case of Daubeney these significant values are, with one exception, positive. It thus appears that the lower rates of seeding adopted for this variety have in fact been consistently below the optimum. The response of Daubeney to increases in the rate of seeding is, however, by no means the same at the different stations. The largest values of b are those

obtained at Charlottetown, where on the average the yield has been augmented by something more than 3 bushels per acre by each increase of $\frac{1}{2}$ bushel in the seed rate within the range studied. At Scott, on the other hand, the maximum value of b is 0.682, obtained in 1926, the values in 1924 and 1925 being quite insignificant. Macdonald College and the University of Alberta occupy a position intermediate between these two extremes. The differences in the value of b obtained from year to year at the same place are in general much less than the differences between places discussed above, though this statement must be modified in its application to Macdonald College. Here in 1924 the lower seed rates gave significantly better results, whereas in the remaining two years the position was reversed.

The results obtained with the other two varieties are subject to considerable fluctuation. On the whole, Banner shows a tendency towards increased yield with increased seed rate, and Abundance a tendency towards decreased yield with increased seed rate. As in the case of Daubeney, however, there are marked differences in the results at the four stations. Here again Scott and Charlottetown present the two extreme conditions. At Scott, both varieties gave negative values of b in all three years; at Charlottetown the values of b are all positive. At Macdonald College there would appear to be a definite tendency for Banner to give higher yields with increased seed-rate. The two years' results here available in the case of Abundance are quite inconclusive. At the University of Alberta, each of the three years has produced results differing both in magnitude and in nature from those of the other two. In 1924, the yield of Abundance was on the average increased by increasing the seed rate, whilst that of Banner was decreased. In 1925 and 1926 this situation was reversed. The observed differences in yield were in each case, however, of a very irregular nature. As a consequence, the standard errors of the regression coefficients are in all cases so large as to destroy their significance.

Instead of calculating a separate regression coefficient for each year, we may estimate the average regression of yield on rate of seeding for all three years. This has been done, and the resulting values of b are shown in Table XII.

TABLE XII
AVERAGE INCREASE IN YIELD (BUSHELS PER ACRE)
FOR EACH HALF-BUSHEL INCREASE IN SEED RATE.
AVERAGE OF THREE YEARS' RESULTS

	Abundance	Banner	Daubeney
Charlottetown	0.657	1.491	2.894*
Macdonald College	-0.172	0.595*	1.052
Scott	-1.004	-0.739	0.226
University of Alberta	-0.486	-0.728	1.958*

When the results of the three years were averaged in this way, it was found that in the case of the variety Abundance at Scott Experimental Station, and Daubeney at Macdonald College, the yield showed a definite tendency to increase with increased seed rate up to a certain

point only. After this, any further increases in rate of seeding not only produced no further increase in yield, but actually exercised a depressive effect. It is therefore not surprising that no significant value of the regression co-

efficient was obtained when a straight line was fitted to the data. When a parabolic curve of the form

$$(y - \bar{y}) = b(x - \bar{x}) + c(x^2 - \bar{x}^2)$$

was fitted, however, significant values were obtained for the parameter c . The actual values were:

Scott, Abundance	$b = 0.149,$	$c = -0.670^*$
Macdonald College, Daubeney	$b = 1.052,$	$c = -1.009^*$

Tendencies of a similar nature were also to be perceived in the results of certain other varieties and stations, most notably Banner at Macdonald College; but in no case could the value of the quadratic coefficient c obtained be regarded as significant.

Of the twelve combinations of variety and station, therefore, five have given significant results when the three-year averages are considered. The nature of these results is indicated in Fig. 3. The average yields of all the check plots of any particular variety grown at any particular station have been taken as starting points; then, using the coefficients of Table XII, except in the two cases where a significant quadratic term was obtained, the curves of regression have been constructed.

The first point to be noted in a consideration of the results portrayed in Fig. 3 is that when all the varieties are sown at rates in the vicinity of their respective optima, the order of yield at Charlottetown, Macdonald College, and the University of Alberta is: Banner, Daubeney, Abundance; though at the two latter stations the differences between Daubeney and Abundance cannot be regarded as significant. At Scott however the order is: Banner, Abundance, Daubeney. The second point is that at Macdonald College, the University of Alberta, and Charlottetown, the relative positions of Daubeney and Abundance could be reversed by a suitable choice of seed rates. At Scott, the differences are not so striking, but their effect upon the comparison of Abundance with the other two varieties is by no means negligible.

Banner has consistently outyielded the other two varieties, at all stations and at all rates of seeding. Its superiority is more marked at the University of Alberta than at any of the other stations. In fact, it would appear that had higher rates of seeding Daubeney been employed at Charlottetown the difference in yield between this variety and Banner might have been reduced to quite insignificant proportions.

We may therefore conclude, from our examination of the data thus far, that variations in the rate of seeding have perceptibly influenced the relative yielding capacities of the three experimental varieties at all four stations; also, that even when the optimum seed rates are employed in each case, the relative performance of the three varieties is by no means the same at all stations. Whilst there can be little doubt that the observed variations are indicative of a real effect, it is nevertheless necessary to keep in mind the considerations developed in Section 4 of this paper. Significant differences in yield between plots sown at different rates have indeed been demonstrated; but we cannot disregard entirely the possibility that a sensible portion of this variation may in fact be attributable to differences in the fertility of the various plots.

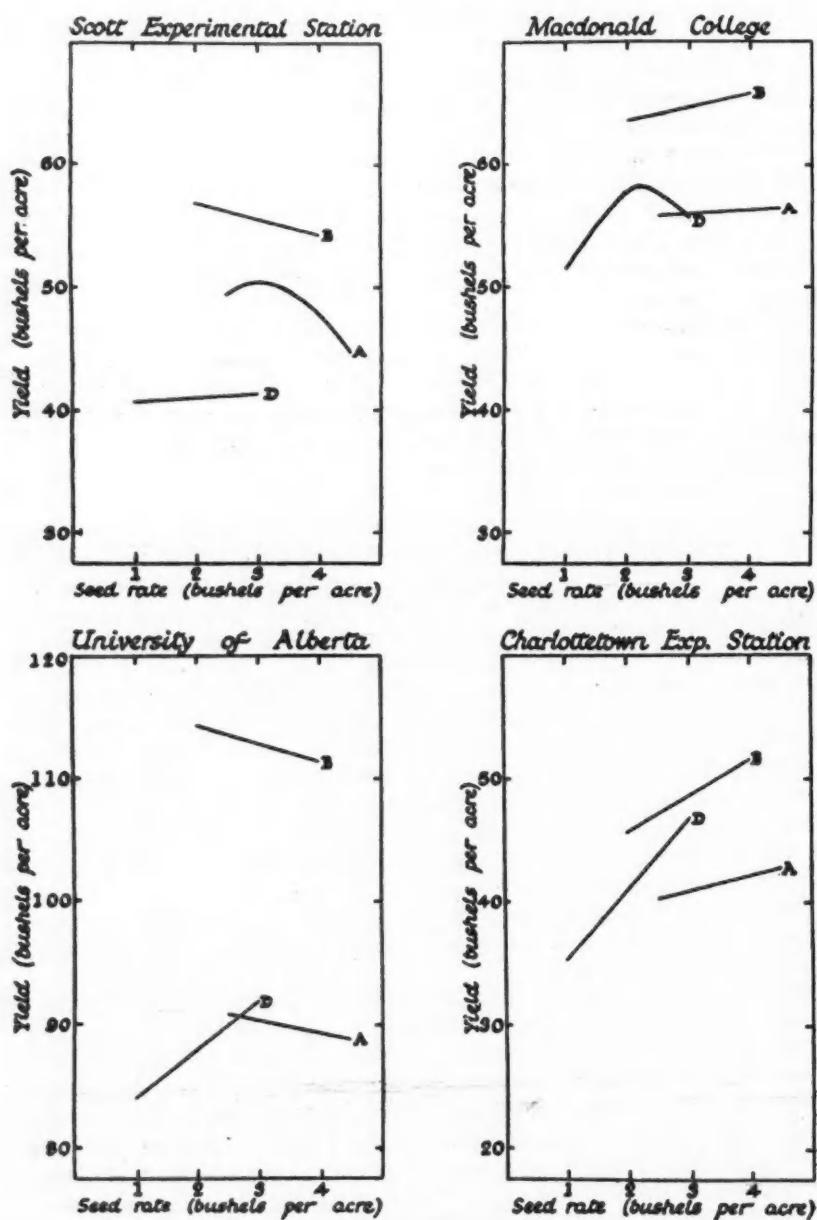


FIG. 3. Curves of regression of yield on rate of seeding. Average of three years' results. A = Abundance, B = Banner, D = Daubeny.

Having demonstrated differences in the behavior of these varieties as regards yield, we may next proceed to enquire to what extent these differences are to be associated with differences in such characteristics as size of seed and tillering capacity. Obviously the first step in any such enquiry is to ascertain the nature and magnitude of the differences which are to be observed between the three varieties in these respects.

TABLE XIII
WEIGHT PER 1000 KERNELS, IN GRAMS, OF THE SEED OF THE THREE VARIETIES USED

Year	Station	Abundance	Banner	Daubeney
1924	Charlottetown	39.7	34.4	21.2
	Macdonald College	39.0	34.2	22.8
	Scott	39.0	34.2	22.8
	Univ. of Alberta	39.0	34.2	22.8
1925	Charlottetown	32.2	30.1	20.7
	Macdonald College	...	29.3	23.0
	Scott	36.4	30.8	21.9
	Univ. of Alberta	28.7	29.1	21.1
1926	Charlottetown	33.6	33.6	25.2
	Macdonald College	33.6	33.6	25.2
	Scott	33.6	33.6	25.2
	Univ. of Alberta	33.5	35.5	25.5

Table XIII shows the estimates of the weight per 1000 kernels of the seed of the different varieties used at each station in the years 1924, 1925 and 1926. The agreement between the figures supplied by the Dominion Cerealists and those obtained by the different stations was on the whole good, except in the case of the variety Abundance in 1925. In this case, discordant results, ranging from 28.7 gm. at the University of Alberta to 36.4 gm. at Scott Experimental Station, were obtained. Apart from this instance, however, there is no reason to suppose that the various lots of seed used in any one year were other than homogeneous.

When the differences between varieties are examined, it appears that in each year the seed of Daubeney had a considerably lower weight per 1000 kernels than that of either Abundance or Banner. The differences in this respect between the two latter varieties are however in all cases small, and, except in 1924, quite insignificant. Considering the yield figures in the light of these observations, we might be led to conclude that the optimum concentration of plants is greater in the case of Daubeney than in the case of the other two varieties, at all stations except Macdonald College. The matter is much complicated, however, by the fact that the stand of plants actually obtained bears by no means a constant relation to the number of seeds planted. A study of the original data shows that considerable fluctuation in the observed stand of plants occurred not only from year to year and from place to place, but also amongst adjacent plots.

There is also, unfortunately, good reason to suspect that an appreciable proportion of this fluctuation is due to certain errors inseparable from the estimation of the actual stand.

The method of estimation adopted has already been described. At harvest time, the centre row of each nine-row plot was dug up in its entirety and the number of individual plants and the total number of tillers (culms) counted. The counting of the total number of culms is of course quite straightforward; but that of the individual plants is often a matter of very considerable difficulty. Groups of plants are often found crowded very closely together, their root systems inextricably intermingled; and any attempt to disentangle them may, and undoubtedly often does, lead either to incomplete separation, on the one hand, or to the pulling apart of tillers, on the other. In the first case two or more individuals are in fact only counted as one, whilst in the second, one actual individual may give rise to a "count" of two or three.

It may be suggested that these two opposing errors are likely to occur with approximately equal frequency in the counting of a given plot, but this does not appear to be the case. The difficulty is most acute where the plants grow most luxuriantly, as on the highly fertile soil at the University of Alberta. Here in 1924 no fewer than 39 counts of plants in excess of the number of germinable seeds sown were recorded; the "percentage stand" being in one case 149. On the other hand, it is by no means uncommon to find that, in the case of two adjacent plots the percentage stand of one is estimated at 110 or so and that of its neighbor at only 60 or 70.

It is of course obvious that these errors affect the estimates of both percentage stand and degree of tillering, though in an opposite sense. Thus, the adherence of two or more plants results in an underestimate of the stand of plants and an overestimate of the degree of tillering; whilst the breaking apart of tillers belonging to the same plant causes an overestimate of stand and an underestimate of tillering.

In addition to errors of this kind affecting the actual operation of counting, another and quite distinct source of inaccuracy in the results arises out of the fact that the centre row of each plot may be by no means representative of the plot as a whole. We have therefore *errors of sampling* as well as errors in the procedure applied to the sample. These would of course persist no matter how precisely the actual counting was performed, and since only one sample was taken from each plot, their magnitude is incapable of determination.

Although the estimate of stand obtained from any individual plot is therefore of doubtful value, yet when the numerous such estimates made in the case of each variety at each station each year are considered in the aggregate, certain definite tendencies reveal themselves. This is illustrated in Table XIV, where the average percentage stand of the different varieties at each station during the three years is to be found.

The values in this table are the average of 64 actual determinations, except in some cases in which the total number of plots was not grown or for other reasons counts were not available. They may therefore be considered to give rather more than a mere indication of the general order of the stand of plants actually obtained; for the combination of a large number of observations, though each is subject to such errors as to render them individually of little value, produces an estimate of considerably greater precision. The results

obtained at Scott have been included in the table, but in making comparisons it must be remembered that in 1925 the counts at this station were made immediately after the emergence of the plants in the spring.

TABLE XIV
AVERAGE STAND OF PLANTS, EXPRESSED AS A PERCENTAGE OF THE NUMBER OF
GERMINABLE SEEDS SOWN

Year ^a	Station	Abundance	Banner	Daubeney
1924	Charlottetown	81.3	89.0	85.3
	Macdonald College	54.1	52.1	55.0
	Scott	88.2	84.2	87.3
	Univ. of Alberta	86.9	95.3	88.1
1925	Charlottetown	81.1	71.2	68.2
	Macdonald College	...	61.4	73.3
	Scott	91.1	94.0	93.2
	Univ. of Alberta	81.7	85.5	83.2
1926	Charlottetown	76.2	75.7	71.8
	Macdonald College	57.5	58.8	58.2
	Scott	74.5	70.1	66.8
	Univ. of Alberta	87.6	86.1	79.6

The most obvious feature of Table XIV is the uniformly low stands of plants obtained at Macdonald College, and the uniformly high ones obtained at the University of Alberta. The effect of the spring count at Scott in 1925 is also very apparent, although it is of course impossible to assert that the large values observed are due entirely to that cause. Counts of the number of plants which emerged in the spring and those surviving to maturity, carried out at all four stations, might have produced results of some interest.

The differences to be observed between varieties, or between the results of the three years, are by no means so distinct. Their precise nature may best be arrived at by an analysis of the variance of the percentage stand figures. Unfortunately, when this is undertaken it is necessary to discard the whole of the results obtained at Scott. A further difficulty arises from the fact that the variety Abundance was not grown at Macdonald College in 1925. In this case however it is possible to estimate a "most likely value" of the missing quantity, using the method developed by Allen and Wishart (1) for the estimation of the yield of a missing plot in field experiments.

This value is estimated from the results obtained at the other stations, the comparative performance of Abundance at Macdonald College in the two other years, and the observed stand of the two other varieties at that station in the year in question. This estimated value is not designed to supply any new information, but simply to fill in the gap and render the data amenable to the processes of the Analysis of Variance, so that the information contained in the observations which actually were made may be extracted. This is the only justification for its employment in the present connection. The present instance well illustrates the undesirable effect which can be produced in experiments of this nature by deviations from uniformity of procedure.

TABLE XV
ANALYSIS OF VARIANCE OF AVERAGE PERCENTAGE STAND

Variance due to	Degrees of freedom	Sum of squares	Mean square	Z
Years	2	76.28	38.14	2.1002*
Places	2	3,625.64	1812.82	
Varieties	2	12.93	6.47	
Interaction variety and place	4	135.46	33.87	0.6330
Interaction variety and year	4	59.62	14.90	
Interaction place and year	4	385.51	96.38	
Remainder	7	190.23	27.18	
Total	25	4,485.66		

The analysis of variance takes the form shown in Table XV. Of the various "mean squares" obtained only two are appreciably larger than the "remainder". They are the ones attributable to general differences between stations and to the interactions of place and year respectively. The values of Z appropriate to these are given in the last column of the table. The first of these is such as would be expected by chance less than once in 100 trials; the second, however, cannot be regarded as significant. We therefore conclude that the only systematic differences of any significance are those between the average results, over the whole three years, obtained at the various stations.

The foregoing analysis deals only with the average stand of all the plots of any one variety grown at each station each year. It is however obviously a matter of some interest to inquire into the possibility of differences in percentage stand between plots of the same variety sown at different rates. This may be attempted by utilizing the averages of the counts of plants made on the centre rows of the eight replicate plots of each seed rate, and the corresponding averages derived from the eight adjacent check plots. Knowing the actual number of germinable seeds sown, and the actual number of plants counted, a series of 2 by 2 tables of the following form may be constructed.

	Produced plants	Failed to produce plants	Total seeds sown
Check	160	143	303
Rate	111	91	202
Total	271	234	505

If the percentage stand was in fact the same in both sets of plots we should expect the quantities 160, 143, and 111, 91, to be in proportion. The question as to whether there is a significant difference in stand therefore resolves it-

self into the question as to whether the observed frequencies differ significantly from the "proportional" values calculated from the marginal totals. The usual procedure in such cases is to calculate the quantity $\chi^2 = S \left(\frac{(x-m)^2}{m} \right)$, where x is the number observed, m the number expected, in this case on the assumption

that the percentage stand is the same in both check and rate plots, and S denotes summation over the four "cells" of the table. In this instance, however, we shall find it advantageous to use the quantity $\sqrt{\chi^2}$, with the convention that when the proportion of plants is higher in the check plots than in the rate plots, $\sqrt{\chi^2}$ shall be taken as positive, and when the proportion of plants is higher in the rate plots than in the check plots, as negative.

It has been shown (3) that when the two sets of observations are in fact random samples from a homogeneous normal population, the quantity $\sqrt{\chi^2}$ will be normally distributed about zero with unit standard deviation. Consequently, if a value of $\sqrt{\chi^2}$ exceeding 2 in absolute magnitude is obtained, there is good reason to believe that the observed proportion of plants maturing, or percentage stand, is not the same in the two sets of plots.

The actual values of $\sqrt{\chi^2}$ obtained in this way are set forth in Table XVI. There are two gaps in the table. The first of these is due to the variety Abundance not being grown at Macdonald College in 1925. The second is due to the fact that at the University of Alberta in 1924 the average number of plants counted in the 2 and 2½ bushel plots of the variety Banner was greater than the number of germinable seeds sown. The pulling apart of tillers of the same plant during the counting process, and errors in seeding may both have contributed in some measure to this result.

Of the 138 values of $\sqrt{\chi^2}$, no fewer than 34 exceed 2.000 in magnitude. There is thus no doubt of the significance of the differences in percentage stand recorded in the case of adjacent check and rate plots. It is necessary to bear in mind however that for reasons already made clear, the numbers of plants recorded may differ to a greater or less extent from the numbers actually growing in the various plots; and the results of Table XVI must therefore be interpreted with some reserve. Nevertheless, it does appear that the percentage stand in the check and rate plots cannot be regarded as uniform.

There is a general tendency (to which numerous exceptions are to be noted) for negative values of $\sqrt{\chi^2}$ to be associated with comparisons in which the rate plot is seeded at a lower rate than the check, and positive values to be associated with those in which the situation is reversed. This is quite understandable. The counts being made in the fall, might indeed be expected to exhibit some evidence of the more intense competition prevailing in the more heavily seeded plots.

It will be observed that the varieties Abundance and Daubeney at Scott Experimental Station in 1925 are amongst the exceptions to this tendency. This also was to be expected, since the effects of plant competition would hardly be apparent at the time (spring) when these counts were made. In the case of Banner, the lower rates of seeding do actually give negative values and the higher rates of seeding positive ones, though none of these are significant.

The largest values of $\sqrt{\chi^2}$, and hence the greatest differences in percentage stand, are to be found associated with the variety Daubeney. Of the four stations, the University of Alberta and Macdonald College exhibit the most pronounced signs of variability in this respect.

TABLE XVI
SIGNIFICANCE OF DIFFERENCES IN PERCENTAGE STAND BETWEEN PLOTS SOWN AT DIFFERENT SEED RATES. VALUES OF $\sqrt{\chi^2}$

	Abundance				Banner				Daubeney			
	2½ bu.	3½ bu.	4 bu.	4½ bu.	2 bu.	2½ bu.	3½ bu.	4 bu.	1 bu.	1½ bu.	2½ bu.	3 bu.
Charlottetown 1924	0.712	1.385	2.719	1.430	0.173	-0.281	0.757	-0.302	-0.704	-1.158	0.647	1.996
1925	0.990	0.544	0.126	2.793	-0.100	1.308	1.401	2.004	-0.327	2.983	1.322	1.998
1926	2.201	-0.574	1.361	3.128	-0.122	1.382	-2.364	1.778	2.158	-0.156	1.624	2.141
Macdonald 1924	-0.697	-1.767	-0.444	-2.011	-0.593	1.445	0.977	0.308	-2.166	-1.707	-1.313	-2.801
1925	-3.347	-0.690	0.616	2.830	-3.557	-0.907	2.623	6.306
1926	-2.153	-0.978	0.407	2.684	-0.473	1.129	6.259	3.847	-2.107	-1.133	0.491	1.543
Scott 1924	-1.460	-1.828	0.632	0.715*	0.158	-0.349	0.266	0.362	0.658	-1.157	1.712	2.658
1925 ^a	-0.152	2.242	0.489	0.653	-0.430	-0.550	0.656	0.729	0.742	0.836	0.502	-2.469
1926	-0.409	-0.438	-0.701	0.888	-1.185	-0.937	1.045	-0.849	0.690	-1.290	-2.484	-1.567
University 1924	-0.628	1.169	0.000	1.138	3.766	3.582	-4.042	0.220	5.127	7.710
1925	0.071	0.171	-0.076	0.960	0.264	0.000	0.516	0.182	-0.902	-0.455	-0.583	0.818
1926	-2.890	-0.941	1.299	2.326	-0.838	-0.686	3.366	3.701	-1.456	-0.264	2.415	0.443

* In 1924, 4½ bu.

TABLE XVII

THE INCIDENCE OF TILLERING. AVERAGE NUMBER OF TILLERS PER PLANT

Year	Station	Abundance	Banner	Daubeney
1924	Charlottetown	1.04	1.03	1.33
	Macdonald College	1.29	1.19	1.45
	Scott	1.01	1.03	1.07
	Univ. of Alberta	1.16	1.01	2.37
1925	Charlottetown	1.16	1.23	1.53
	Macdonald College	1.01	1.28	1.69
	Univ. of Alberta	1.01	1.03	1.49
1926	Charlottetown	1.10	1.04	1.70
	Macdonald College	1.40	1.46	1.86
	Scott	1.24	1.21	1.36
	Univ. of Alberta	1.00	1.02	1.58

Differences in the degree of tillering of the three varieties may be examined in precisely the same way as were differences in the average percentage stand. Table XVII gives the average number of tillers per plant appropriate to the different varieties, stations and years. Table XVIII shows the results of an analysis of the variance of the values obtained at Charlottetown Experimental Station, Macdonald College, and the University of Alberta.

TABLE XVIII

ANALYSIS OF VARIANCE OF AVERAGE NUMBER OF TILLERS PER PLANT

	Degrees of freedom	Sum of squares	Mean square	Z
Years	2	0.0107	0.0054	
Places	2	0.1845	0.0922	0.3783
Varieties	2	1.5829	0.7915	1.4653*
Interaction variety and place	4	0.2119	0.0530	
Interaction variety and year	4	0.0515	0.0129	
Interaction place and year	4	0.3581	0.0895	
Remainder	7	0.2957	0.0422	
Total	25	2.6953		

It is apparent from Table XVII that the variety Daubeney has consistently produced more tillers per plant than either Abundance or Banner. Except at Macdonald College, and at Scott in 1926, the tillering of the two latter is in fact very slight.

This varietal effect is the only one to appear significant in the analysis of variance. It is moreover possible to carry the analysis one stage further than that indicated in Table XVIII. The sum of squares appropriate to varietal differences may be subdivided into two portions, one due to the differences between Daubeney and the mean of the other two varieties, and the other due to differences between the two latter. When this is done, it is found that of the sum of squares, 1.5829, no less than 1.5811 is attributable to the differences

between Daubeney and the mean of Abundance and Banner, and only 0.0018 to differences between these two. Thus the only significant feature of the tillering figures is that Daubeney has consistently tillered more freely than the other two varieties, the differences in performance between Abundance and Banner being quite negligible.

It might be thought desirable to attempt an examination into the effect of thickness of stand on tillering in the case of the different varieties; and precise information on this point would indeed have been of some interest. Upon a consideration of the nature of the errors to which the observations are subject, however, the present data do not appear suitable for this purpose. It has already been pointed out that any underestimate of the stand of plants is automatically accompanied by an overestimate of the amount of tillering, and *vice versa*. In this way a certain spurious correlation is introduced into the observed results. Since there is reason to believe that errors of this kind of some magnitude have in fact occurred, conclusions of any exactitude would seem to be rendered impossible.

We may, however, construct a table, such as Table XIX, showing the average number of tillers per plant observed in the plots devoted to the different seed rates. Of course the stands of plants in the individual replicate plots may vary considerably amongst themselves, but the average stand of plants in, for example, the plots sown at the rate of 4 bushels per acre will in general be distinctly denser than that in the plots of the same variety sown at the rate of $3\frac{1}{2}$ bushels per acre.

The effect of increased density of plants in reducing the amount of tillering is apparent at all stations except Macdonald College, where the results are so conspicuously irregular as to be in a class by themselves. There is little to choose between the results obtained with Abundance and Banner; and certainly nothing to suggest that the former variety is in any way inherently inferior in tillering capacity. The variety Daubeney on the other hand shows not only a higher average number of tillers per plant, as appears from the preceding analysis of variance, but also a much greater flexibility in tillering at the different seed rates. This is particularly marked at the University of Alberta, where the high fertility of the soil doubtless facilitated increased tillering with thinner seeding.

Besides the evidently greater inherent tillering capacity of Daubeney, two other circumstances may have contributed in some degree to its greater range of tillering. In the first place, the seed of this variety having a much lower weight per 1000 kernels than that of Abundance and Banner, the range from the highest to the lowest number of seeds sown will be correspondingly greater. So, presumably, will the range of density of the plant population, especially in the earlier part of the growing season, when tillering would be taking place. By the fall, when the plant counts are made, the different intensities of plant competition may be expected to have modified this situation somewhat; and it will be remembered that the largest values of $\sqrt{\chi^2}$ in Table XVI are in fact associated with the variety Daubeney. The second point is that the lowest

TABLE XIX
AVERAGE NUMBER OF TILLERS PER PLANT AT VARIOUS SEED RATES

	Abundance					Banner					Daubeney					
	2½ bu.	3 bu.	3½ bu.	4 bu.	4½ bu.	2 bu.	2½ bu.	3 bu.	3½ bu.	4 bu.	1 bu.	1½ bu.	2 bu.	2½ bu.	3 bu.	
Charlottetown	1924	1.06	1.04	1.04	1.02	1.02	1.04	1.04	1.03	1.02	1.02	1.54	1.49	1.32	1.18	1.20
	1925	1.25	1.15	1.15	1.13	1.24	1.22	1.31	1.22	1.24	1.21	1.56	1.52	1.51	1.50	1.58
	1926	1.63	1.02	1.02	1.02	1.02	1.04	1.01	1.05	1.01	1.02	2.61	1.83	1.77	1.60	1.46
Macdonald College	1924	1.34	1.27	1.14	1.17	1.10	1.34	1.13	1.17	1.14	1.22	1.94	1.30	1.39	1.23	1.27
	1925	1.13	1.34	1.31	1.33	1.57	1.88	1.91	1.63	1.54	1.82
	1926	1.38	1.41	1.32	1.42	1.51	1.78	1.46	1.26	1.59	1.67	1.90	2.43	1.75	1.69	1.71
Scott	1924	1.01	1.01	1.00	1.00	1.00	1.12	1.03	1.01	1.00	1.00	1.39	1.05	1.02	1.01	1.01
	1925
	1926	1.15	1.25	1.18	1.17	1.37	1.23	1.27	1.23	1.17	1.11	2.09	1.38	1.31	1.09	1.11
University of Alberta	1924	1.30	1.18	1.13	1.09	1.06	1.04	1.02	1.01	1.01	1.00	3.74	2.79	2.14	2.02	1.89
	1925	1.04	1.01	1.00	1.00	1.01	1.16	1.06	1.01	1.00	1.00	2.36	1.70	1.40	1.16	1.14
	1926	1.01	1.01	1.00	1.00	1.00	1.06	1.04	1.01	1.02	1.00	2.48	1.95	1.48	1.20	1.10

seed rate of Daubeney is considerably below that of the other two varieties, the conditions for tillering thus being made more favorable.

We may next consider to what extent the observed differences in weight per 1000 kernels of seed and in tillering capacity, and the fluctuations in percentage stand have affected the yield of the three varieties.

Reference to Tables XI and XII at once reveals that the variety Daubeney, which has shown the greatest tillering capacity and might therefore be expected to adjust itself more readily to different rates of seeding, has on the contrary shown the most pronounced variation in yield. This situation may perhaps be to some extent accounted for by the considerations advanced in discussing the tillering results. The smaller weight per 1000 kernels of this variety results in a much wider range of plant densities, whilst the lower rates of seeding are definitely below that optimum range within which variations in the seed rate appear to have very little effect on yield. The greater degree of tillering in evidence at the lower seed rates is apparently not sufficient to offset the reduction in yield caused by the diminution of the plant population.

The effect of differences in percentage stand would seem to require some investigation, in view of the considerable fluctuation in this respect observed from plot to plot. The obvious method to adopt would appear to be to determine the regression of yield on plant number, as opposed to seed rate.

There are several ways in which this may be done.

The first to be adopted was that of obtaining separate regression coefficients for each variety, place, and year, as was done in the case of rate of seeding with the results shown in Table XI. In the present instance, however, the values of the regression coefficient were not determined from 4 yield-figures, each the average of 16 differences between adjacent check and rate plots, but from 32 yield-figures, the exact nature of which may best be understood by reference to the accompanying diagram (Fig. 4). Here R_1 , R_2 , and C_1 , C_2 , represent yields of individual three-row check and rate plots, n_1 , n_2 , and n_3 the plants counted in the central rows of the nine-row plots, of which there were eight for each rate of seeding. Then the difference in plant number used in calculation is taken as $n_2 - \frac{n_1 + n_3}{2}$ and the difference in yield as $R_1 + R_2 - C_1 - C_2$, precisely the quantity used in calculating the values of t at the beginning of the analysis. The total number of these differences which may be formed within each variety is 32.

The values of the regression coefficients obtained are given in Table XX. These represent the average increase in yield for each increase in stand, over the stand of the check plots, of 100,000 plants per acre.

An examination of Tables XI and XX reveals that although the actual magnitudes of the various regression coefficients are of course different, their relative order is much the same in the two cases. There are however certain points of dissimilarity worth noting.

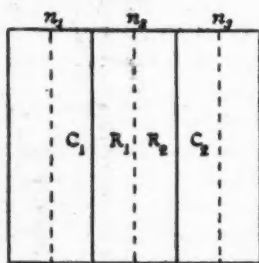


FIG. 4

TABLE XX
AVERAGE INCREASE IN YIELD (BUSHEL PER ACRE), FOR EACH INCREASE OF 100,000
IN THE NUMBER OF PLANTS PER ACRE

		Abundance	Banner	Daubeney
Charlottetown	1924	0.420	0.232	0.884*
	1925	-0.089	2.078*	1.414*
	1926	0.366	0.194	0.880*
Macdonald College	1924	0.005	0.330	-0.330
	1925	...	0.391	1.532*
	1926	0.374	0.233	1.268*
Scott	1924	-0.810*	-0.219	0.026
	1925	-0.110	-0.471	-0.017
	1926	-0.924*	-0.381	0.330
University of Alberta	1924	1.688	-0.114	0.809
	1925	-1.301*	0.094	0.978*
	1926	-0.729	-0.412	0.750*

The total number of coefficients which may be regarded as statistically significant is now 11, as compared with 8 in Table XI. These are indicated in Table XX by an asterisk. Of the 11, 7 are found to belong to the variety Daubeney, which has at all stations excepting Scott shown a definite tendency towards higher yield with increased stand of plants. Furthermore the depression in yield with increasing seed rate, noted in the case of this variety at Macdonald College in 1924, though still in evidence, loses its significance when related to plant number.

Considering now the other two varieties, Abundance has shown a definite tendency to give lower yields from the denser stands of plants in two out of the three years at Scott. At the University of Alberta the results are more irregular, but a significant negative value of the regression coefficient is obtained for 1925. At Macdonald College and Charlottetown no significant tendency in either direction is discernible. The only significant result in the case of Banner is that at Charlottetown in 1925. It may however be more than a coincidence that at Scott negative values of the regression coefficient, and at Macdonald College positive values, were obtained in all three years.

An attempt was next made to develop for each of the 35 combinations of variety, station, and year a quadratic regression equation, such as was obtained in two instances when considering the relation between yield and rate of seeding. The values of the quadratic coefficients resulting were however uniformly insignificant.

This was somewhat disappointing, as it had been hoped by this method to determine the optimum stand of plants of each variety at the different stations. It was realized that the result was probably to a considerable extent attributable to errors in the counts of plants. Nevertheless it was thought desirable to investigate to what extent the observed results might be influenced by variations in the general fertility level of the land on which plots sown at

TABLE XXI
MEAN YIELD IN BUSHEL PER ACRE OF CHECK PLOTS ADJACENT TO PLOTS SOWN AT VARIOUS RATES

	Abundance				Banner				Daubeney			
	Rate plots sown at				Rate plots sown at				Rate plots sown at			
	2½ bu.	3½ bu.	4 bu.	4½ bu.	2 bu.	2½ bu.	3½ bu.	4 bu.	1 bu.	1½ bu.	2½ bu.	3 bu.
Charlottetown 1924	40.65	33.03	31.95	46.55	47.51	53.33	50.43	64.55	33.08	30.96	33.06	40.84
1925	35.91	44.58	52.54	45.26	42.80	43.22	45.25	42.20	33.20	33.13	34.63	38.16
1926	26.04	39.18	43.91	40.83	44.56	47.33	56.90	62.04	64.95	51.57	57.57	61.25
Macdonald	52.34	43.71	41.62	52.02	49.52	54.98	52.58	51.81	42.27	37.22	36.46	50.93
College 1925	60.90	72.97	70.86	72.51	58.78	62.91	68.00	64.82
1926	58.68	66.53	59.19	64.53	72.85	74.08	74.70	71.61	47.52	75.03	68.27	64.49
Scott 1924 ^a	25.77	28.48	24.93	17.43	37.11	32.67	25.07	21.87	32.38	15.10	15.86	12.35
1925	82.84	87.28	83.25	90.20	88.10	88.98	86.90	91.44	72.04	72.65	72.30	70.83
1926	38.75	36.97	33.84	35.01	54.60	56.68	47.24	45.33	35.85	34.55	36.33	31.27
University	92.10	84.82	90.03	112.04	124.09	124.71	132.96	124.35	87.71	84.56	78.82	86.52
of Alberta 1925	94.22	89.32	91.58	97.98	123.55	126.60	113.95	114.15	101.67	106.15	99.97	106.50
1926	89.02	78.14	89.17	89.51	84.38	91.30	88.48	102.68	68.67	73.88	67.64	69.12

different rates were grown. The nature of these variations has been discussed in Section 4, where it was also pointed out that the field arrangement made it possible to demonstrate their existence only in the case of different sets of plots of the same variety.

For this purpose the "range" of plots of each variety grown at any particular station may be subdivided into four portions. These four portions, which consist of all the plots of any one seed-rate and their adjacent check plots (32 three-row plots in all), we may term "blocks". It is with the check plots, of which there are 16 in each block, that we are concerned, for these being sown at a uniform rate, their yields allow us to judge whether the general level of fertility of the various blocks may be regarded as the same or not.

In Table XXI will be found the average yield of the 16 check plots in the blocks containing the replicate plots of the various seed rates. It will be observed that the mean yield varies from block to block, often by a considerable amount. To test the significance of these variations is to test whether the variation between blocks of plots is significantly greater than the variation between plots within the same block.

This resolves itself into a simple analysis of the variance of the yields of the 64 check plots in each range. The sum of the squares of the deviations of the 64 observed yields from their general mean may be divided into a portion representing the deviations of the four block means from the general mean, and a portion representing the deviation of the 16 plot yields in each block from the mean yield of their particular block. When these quantities are divided by the appropriate number of degrees of freedom (namely 3 and 60), mean square deviations, or "variances", "within blocks" and "between blocks" are obtained. The Z test may be used to determine whether the variance between blocks is significantly greater than that within blocks.

Table XXII gives the sums of squares, mean squares, and values of Z obtained by applying this process to the yield figures of each station. The number of degrees of freedom available in each case has also been given, since the full complement of 64 yields was not always available. The last column of Table XXII gives the 1% value of Z , *i.e.*, the value which is expected to be equalled or exceeded by chance only once in 100 times.

It will be observed that in certain instances (for example, Banner and Daubeney at Scott in 1925) no value of Z is given. In these cases, which are few in number, the variance "within blocks" is actually greater than that "between blocks". In the great majority of cases, however, the reverse condition holds, and some of the values of Z obtained are very large indeed.

The differing effects of soil heterogeneity, encountered at the various stations, as revealed by these figures, are of some interest. Thus at Scott in 1924 the arrangement of the plots appears to have coincided to a marked extent with systematic variations in soil fertility (see Table XXI); and values of Z in the neighborhood of 2.000 are obtained, whereas the 1% value is only 0.7100. In 1925 however the variation between blocks is of a very much lower order. Two of the varieties indeed gave a variance "within blocks" greater than that "between blocks". In 1926 the variance between blocks was more

TABLE XXII
ANALYSIS OF VARIANCE OF CHECK PLOT YIELDS

Variety	Variance	Degrees of freedom	Sum of squares	Mean square	Z	1% Z
Charlottetown 1924 Abundance	Between blocks	3	2190.23	730.08		
	Within "	59	2560.02	43.39	1.4124	0.7100
	Between blocks	3	2546.77	848.92		
	Within "	59	5177.08	87.75	1.1462	0.7100
	Between blocks	3	867.88	289.29		
	Within "	59	1236.08	20.95	1.3127	0.7100
Charlottetown 1925 Abundance	Between blocks	3	2147.66	715.88		
	Within "	58	2327.95	40.14	1.4406	0.7114
	Between blocks	3	83.77	27.92		
	Within "	60	5951.47	99.19
	Between blocks	3	266.09	88.70		
	Within "	60	2609.63	43.49	0.3564	0.7086
Charlottetown 1926 Abundance	Between blocks	3	2980.17	993.39		
	Within "	60	2888.35	48.14	1.5136	0.7086
	Between blocks	3	3200.90	1066.97		
	Within "	60	4214.04	70.23	1.3604	0.7086
	Between blocks	3	1561.28	520.42		
	Within "	60	3802.13	63.37	1.0028	0.7086
Macdonald College 1924 Abundance	Between blocks	3	1483.22	494.41		
	Within "	60	2290.55	38.18	1.2806	0.7086
	Between blocks	3	235.37	78.46		
	Within "	60	3190.69	53.18	0.1944	0.7086
	Between blocks	3	2130.22	710.07		
	Within "	60	2638.39	43.97	1.3910	0.7086
Macdonald College 1925 Banner	Between blocks	3	1549.01	516.34		
	Within "	60	2875.18	47.92	1.1886	0.7086
	Between blocks	3	712.36	237.45		
	Within "	60	2408.86	40.15	0.8881	0.7086
Macdonald College 1926 Abundance	Between blocks	3	660.79	220.26		
	Within "	55	2381.82	43.31	0.8133	0.7160
	Between blocks	3	87.35	29.12		
	Within "	50	1103.62	22.07	0.1386	0.7234
	Between blocks	3	5426.90	1809.00		
	Within "	60	6531.06	108.85	1.4053	0.7086

TABLE XXII—Continued

Variety	Variance	Degrees of freedom	Sum of squares	Mean square	Z	1% Z
Scott 1924	Abundance	Between blocks 3	1052.51	350.84		
		Within " 59	690.15	11.70	1.7003	0.7100
	Banner	Between blocks 3	2272.05	757.35		
		Within " 59	1440.25	24.41	1.7174	0.7100
	Daubeney	Between blocks 3	3839.96	1279.99		
		Within " 59	934.72	15.84	2.1960	0.7100
Scott 1925	Abundance	Between blocks 3	570.11	190.04		
		Within " 59	3692.83	62.59	0.5502	0.7100
	Banner	Between blocks 3	170.27	56.76		
		Within " 59	3428.80	58.12
	Daubeney	Between blocks 3	15.79	5.26		
		Within " 59	4782.50	81.06
Scott 1926	Abundance	Between blocks 3	223.36	74.45		
		Within " 59	1020.24	17.29	0.7300	0.7100
	Banner	Between blocks 3	1429.81	476.60		
		Within " 59	1981.82	33.59	1.3262	0.7100
	Daubeney	Between blocks 3	239.20	79.73		
		Within " 59	1304.36	22.11	0.6414	0.7100
University of Alberta 1924	Abundance	Between blocks 3	4049.20	1349.73		
		Within " 44	17315.78	393.54	0.6169	0.7306
	Banner	Between blocks 3	823.98	274.66		
		Within " 55	17188.56	312.52
	Daubeney	Between blocks 3	745.83	248.61		
		Within " 60	4472.17	74.54	0.6022	0.7086
University of Alberta 1925	Abundance	Between blocks 3	642.35	214.12		
		Within " 59	11980.52	203.06	0.0252	0.7100
	Banner	Between blocks 3	2020.94	673.65		
		Within " 60	3028.11	50.47	1.2912	0.7086
	Daubeney	Between blocks 3	500.42	166.81		
		Within " 59	4323.47	73.27	0.4613	0.7100
University of Alberta 1926	Abundance	Between blocks 3	1479.22	493.07		
		Within " 60	3438.06	57.30	1.0762	0.7086
	Banner	Between blocks 3	2832.67	944.22		
		Within " 59	3173.62	53.79	1.4327	0.7100
	Daubeney	Between blocks 3	368.09	122.70		
		Within " 60	2091.01	34.85	0.6294	0.7086

pronounced again, though the values of Z obtained are not so great as those of 1924. At Macdonald College, on the other hand, evidence of considerable variation between blocks of similarly treated plots is apparent in all three years. Various other comparisons may be made from the results of Tables XXI and XXII, a study of which should be instructive for those interested in the problems of field experimental technique.

There can be little doubt, from these results, that at each of the stations certain comparisons between different rates of seeding of the same variety have been made on land differing significantly in general fertility level. The design of the experiment does not allow of any investigation into the possible differences in fertility between the areas on which different varieties were grown. In view of the results obtained in the foregoing analysis, however, the existence of these can hardly be regarded as unlikely.

Some of the possible effects of these soil differences upon the yield comparisons obtained have already been suggested. The question which naturally arises therefore is: whether some method cannot be employed to estimate the magnitude of any such effects and to eliminate this disturbing element from the experimental results.

An attempt was made to achieve this end, using the statistical procedure known as *partial regression*. The reasoning which lay behind the particular method adopted was somewhat as follows: It was assumed that the magnitude of the effect of differences in fertility level on the yield comparisons varied to some extent with the rate of seeding employed. Thus, if a higher level of fertility favored the higher rates of seeding, this effect might be expected to be most pronounced in the comparison "highest rate of seeding *vs.* check", and *vice versa*. Further, it was thought that in this connection the results of all three years, for any variety at any particular station, might well be considered together. In this way it was hoped to obtain a sort of average effect of rate of seeding upon yield, corrected for any observed systematic influence of fertility, whether due to soil differences or climatic factors.

A regression equation of the form

$$(y - \bar{y}) = a(x_1 - \bar{x}_1) + b(x_2 - \bar{x}_2) + c(x_3 - \bar{x}_3)$$

was obtained for each variety at each station. Here y represents the differences between the mean yield of the set of 16 plots of any particular seed rate and the mean yield of the adjacent 16 check plots. For each variety at each station there will be four such values of y each year, or twelve in all. The difference in seed rate between the "rate" plots under consideration and the adjacent check plots is represented by x_1 , while x_2 is, simply x_1^2 . The "fertility factor" is represented by x_3 , which is the product of the deviation of the average yield of the 16 check plots in the particular "block" under consideration from the mean of the 12 such averages proper to the particular variety and station and x_1 . Symbolically

$$x_3 = (Y_c - \bar{Y}_c)x_1$$

where Y_c represents the mean yield of the 16 check plots in any particular

block. The mean of the 12 values of y is represented by \bar{y} ; the mean of the 12 values of x_1 , by \bar{x}_1 ; and the mean of the 12 values of Y_c by \bar{Y}_c .

The values of the regression coefficients a , b , and c , appropriate to each variety at each station, were determined by the ordinary Least Squares solution. The results of this calculation are to be found in Table XXIII.

TABLE XXIII

PARTIAL REGRESSION COEFFICIENTS OF YIELD (BUSHEL PER ACRE) ON RATE OF SEEDING (BUSHEL PER ACRE) AND "FERTILITY FACTOR"

Station	Abundance			Banner			Daubeney		
	a	b	c	a	b	c	a	b	c
Charlottetown	.2739	.1056	-.0057	1.5182*	-.3803	-.0685	2.9217*	-.1167	-.0184
Macdonald College	-.2532	.0838	.0337	.5871	-.0457	-.0078	1.1524*	-1.3967*	.1064*
Scott	.4470	-.8840*	.0246	-.5920	-.7009	-.0148	.2260	.1831	.0018
University of Alberta	-.2038	-.1024	.0102	-.7457	-.1086	-.0567	1.9847*	-.4368	.0228

The significance of the various regression coefficients was tested by calculating the value of t , the ratio of the observed coefficient to its estimated standard error, and then determining from a table of "Student's" integral the probability of obtaining such a value of t by chance. Those coefficients which are to be regarded as statistically significant are indicated in Table XXIII by an asterisk.

It will be observed, on referring to the table, that there are altogether 7 significant values. Of these, 5 are associated with the variety Daubeney. Only one of the 12 coefficients c attained to the level of significance. It is however noteworthy that all the values of this coefficient obtained at Charlottetown are negative, whereas at the other stations Abundance and Daubeney give positive, and Banner negative values.

Having obtained the partial regression coefficients of yield on seed rate and on the "fertility factor", it is possible to investigate the relation between yield and seed rate under conditions of constant fertility. Putting then $(x_3 - \bar{x}_3)$ in each case equal to its expected value (which is of course conveniently zero) the theoretical values of y corresponding to different values of x_1 and $x_2 (=x_1^2)$ may be calculated, using the coefficients a and b of Table XXIII. This has been done, with the results shown diagrammatically in Fig. 5.

In considering the results portrayed in Fig. 5, two points should be borne in mind. Firstly the curves represent not the actual results of any particular year or years, but an attempt to estimate, from the experience of all three years, a set of results appropriate to the average condition of fertility obtaining at any particular station. Secondly, of the various regression coefficients a and b , only six may be regarded as differing significantly from zero. The standard

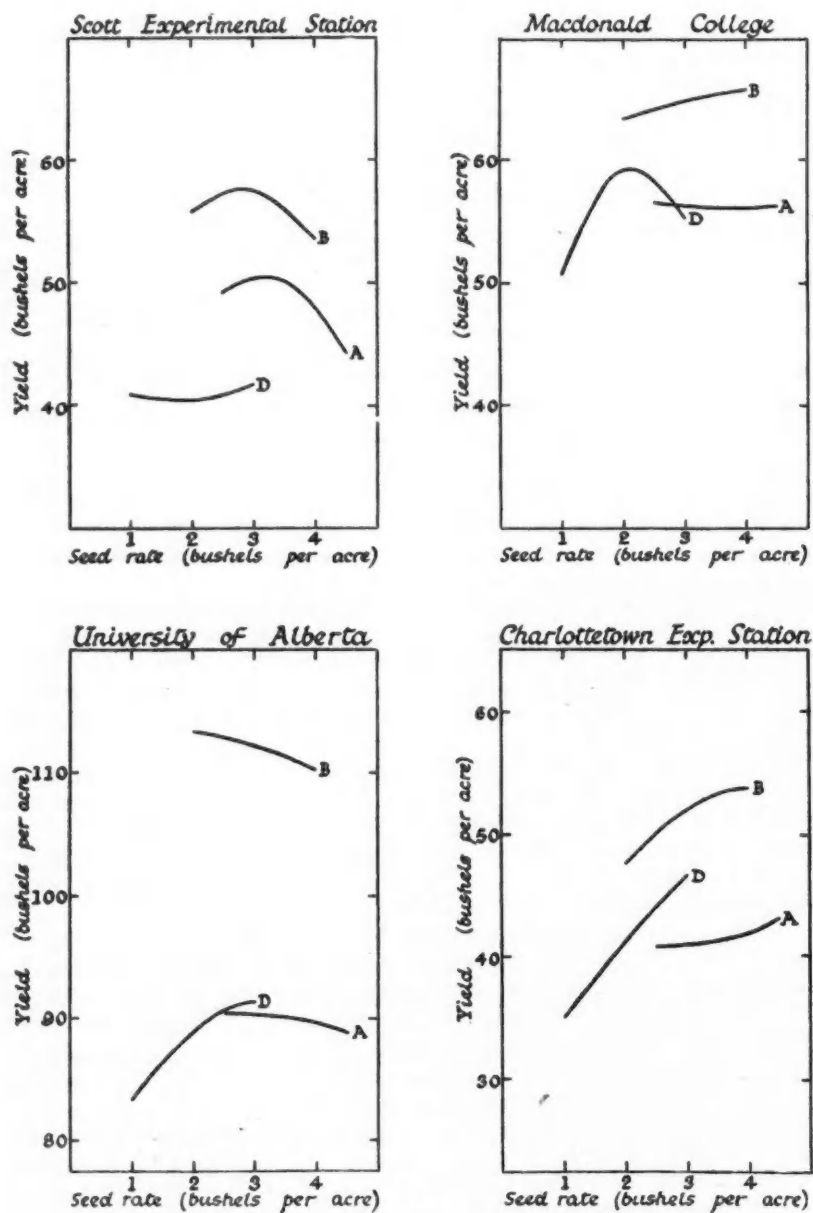


FIG. 5. Curves of partial regression of yield on rate of seeding. Three years' results. A = Abundance, B = Banner, D = Daubeny.

TABLE XXIV
STANDARD ERROR OF ESTIMATION OF
 $(Y - \bar{Y}) = a(x_1 - \bar{x}_1) + b(x_2 - \bar{x}_2) + c(x_3 - \bar{x}_3)$
APPROPRIATE TO THE CURVES OF FIG. 5

Station	Abundance	Banner	Daubeney
Charlottetown	1.69	2.79	2.62
Macdonald College	1.57	1.95	2.02
Scott	1.17	2.48	1.68
Univ. of Alberta	6.07	2.20	1.50

error, with which any particular point on the various curves is estimated, is given in Table XXIV.

The first feature to be noted in Fig. 5, as in Fig. 3, is that the optimum yields of the three varieties are in the order Banner, Daube-

ney, Abundance at Macdonald College, Charlottetown Experimental Station, and the University of Alberta. At the last station, however, the difference in optimum yield between Daubeney and Abundance is quite insignificant. At Scott Experimental Station on the other hand Daubeney gives consistently lower yields than either of the other two varieties.

The response of Daubeney to increases in the rate of seeding at Macdonald College, the University of Alberta, and Charlottetown is also apparent. The absence of any such response at Scott is therefore the more noteworthy. It will however be observed that at this station the other two varieties show a definite tendency towards decreased yields at the higher seed rates. The optimum yield of Daubeney at Macdonald College when sown at approximately two bushels per acre is also clearly evident from the figure.

We may now attempt to consider the effect of these fluctuations in yield upon varietal comparisons. It is apparent that in general the optimum rate of seeding may differ for different varieties at the same station, and certainly does so for the same variety at different stations. In the neighborhood of the optimum, however, the yield is usually only slightly affected by changes in the seed rate. Furthermore, at all stations one variety, or more, exhibits no significant increase or decrease in yield at all over the range of seed rates explored.

These circumstances enable us to pick out a certain definite rate of seeding at each station, at which all three varieties may be sown and yet give an unbiased indication of their relative yielding capacities. Thus when the results of Fig. 5 are considered in the light of the standard error attached to any particular point on the various curves (see Table XXIV), it is at once apparent that at Scott a quite satisfactory comparison results if all varieties be seeded at the rate of 3 bushels per acre. Similarly at Macdonald College there would appear to be little loss of accuracy involved if all varieties were sown at the rate of $2\frac{1}{2}$ bushels per acre. At the University of Alberta, the fluctuations in yield between similarly treated plots of Abundance, and also of Banner, are so large that any effect upon yield comparisons resulting from seeding all plots at the rate of 3 bushels per acre would be relatively negligible. It is not possible to speak with such certainty in the case of Charlottetown, since at this station the maximum yield of Daubeney has clearly not been attained. It is possible however that at a seed rate of 4 or $4\frac{1}{2}$ bushels per acre all varieties

would be yielding sufficiently near to their optimum to enable a satisfactory varietal comparison to be made.

We are thus led to conclude that appropriate to each station there is a certain seed rate at which all three varieties may be sown and yet give yields which are truly comparable; the errors resulting from the varieties not all being seeded at their optimum rates being well within the ordinary experimental error as estimated from the fluctuations in yield of plots of the same variety. The particular seed rate to adopt would appear to vary from station to station, and is probably affected by both soil and climatic conditions.

The similar behavior of Abundance and Banner at any particular station is very apparent. This is interesting in view of the close similarity, previously noted, of these two varieties in size of seed and tillering behavior. As Daubeney differs from the other two varieties in both of these characteristics it is impossible to separate their individual contributions to the observed results. The effect of tillering, however, would presumably be a tendency to stabilize the yield at different seed rates. In the case of Daubeney this effect appears to have been masked by the counter influence of smallness of seed increasing abnormally the range in plant number.

Regression equations of an exactly similar nature were next determined, using however plant number as an independent variable instead of rate of seeding. The procedure was the same as that outlined above, except that x_1 is now the difference between the averages of the eight counts of plant number made on the "check" and "rate" plots of each block.

This undertaking had a twofold objective. First, to investigate the relationship between yield and stand of plants of the three varieties. Second, to see if the deviations from the rate of seeding regression equations producing the larger standard deviations of Table XXIV might not be to some extent explained by fluctuations in the percentage stand of plants obtained.

The values of the regression coefficients a , b , and c , obtained in this way, are given in Table XXV. Those which are to be regarded as differing significantly from zero are indicated by an asterisk. It will be observed that there are again seven such, but that these do not in all cases correspond to the seven significant coefficients of Table XXIII.

The goodness of fit of the two sets of regression formulas may be investigated by calculating the two quantities $SS(Y-y)^2$, where y is an observed yield value and Y the corresponding value calculated from the regression equation. The double summation sign indicates that the sums of the squares of the deviations from the twelve regression equations in each set are themselves to be summed. The resulting quantities are 647.83 in the case of the rate of seeding equations and 694.45 in the case of the plant number equations. The corresponding number of degrees of freedom is in both cases 92. We obtain by division, mean squares of 7.0416 and 7.5484 respectively, and find $Z=0.0347$. This value is quite insignificant. We therefore conclude that, taking the experiment as a whole, the goodness of fit of the two sets of regression equations is sensibly the same. Certainly there is no evidence of any superiority of those employing plant number as opposed to rate of seeding.

TABLE XXV

PARTIAL REGRESSION COEFFICIENTS OF YIELD (BUSHELS PER ACRE) ON PLANT NUMBER (THOUSANDS PER ACRE) AND "FERTILITY FACTOR"

Station	Abundance			Banner			Daubeney		
	a	b	c	a	b	c	a	b	c
Charlottetown	.00001	.00184	-.00594*	.00842*	.00025	-.00641	.01264*	.00310	.00033
Macdonald College	-.00453	.00113	-.00224	.00522	.00140	-.00110	.00965*	-.00338*	.00222
Scott	-.00913	.00035	.00125*	-.00316	-.00150	-.00026	.00112	.00013	.00025
University of Alberta	-.00167	-.00141	.00253	-.00332	-.00274	-.00246	.00877*	-.00071	.00054

As in the preceding case, the coefficients *a* and *b* of Table XXV have been used to construct the curves of Fig. 6. Table XXVI gives the standard error appropriate to any point on the various curves.

TABLE XXVI

STANDARD ERROR OF ESTIMATION OF
 $(Y - \bar{Y}) = a(x_1 - \bar{x}_1) + b(x_2 - \bar{x}_2) + c(x_3 - \bar{x}_3)$
 APPROPRIATE TO THE CURVES OF FIG. 6

Station	Abundance	Banner	Daubeney
Charlottetown	1.65	3.00	2.31
Macdonald College	1.64	2.07	1.60
Scott	2.03	2.34	1.66
Univ. of Alberta	5.84	3.68	1.21

The curves of Fig. 6 differ somewhat from those of Fig. 5. Two circumstances presumably account for this: differences in seed size between varieties, and the fact that the number of seeds sown bears by no means a constant ratio to the final stand of plants obtained.

Although the goodness of fit of the plant number regression equations is not significantly different from that of the rate of seeding equations, one point of considerable interest does emerge from these results. That is, that at three out of the four stations there are indications that the optimum stand of Daubeney, the small-seeded variety, may be greater than that of the other two varieties. At Charlottetown the optimum stand of both Daubeney and Banner does not appear to have been reached. Actually only at the University of Alberta can the observed difference in optimum stand be regarded as significant.

The various errors to which the determination of the plant number is subject have already been discussed. On account of these, the validity of the present results cannot be established above suspicion, even though the values actually used are all averages of eight individual counts. Nevertheless the results, such as they are, do not lend support to the principle of the Swedish system of variety testing noted in Section 1 of this paper.

A third set of regression equations were obtained, using the number of tillers per acre as a variable in place of plant number. The resulting coefficients are shown in Table XXVII.

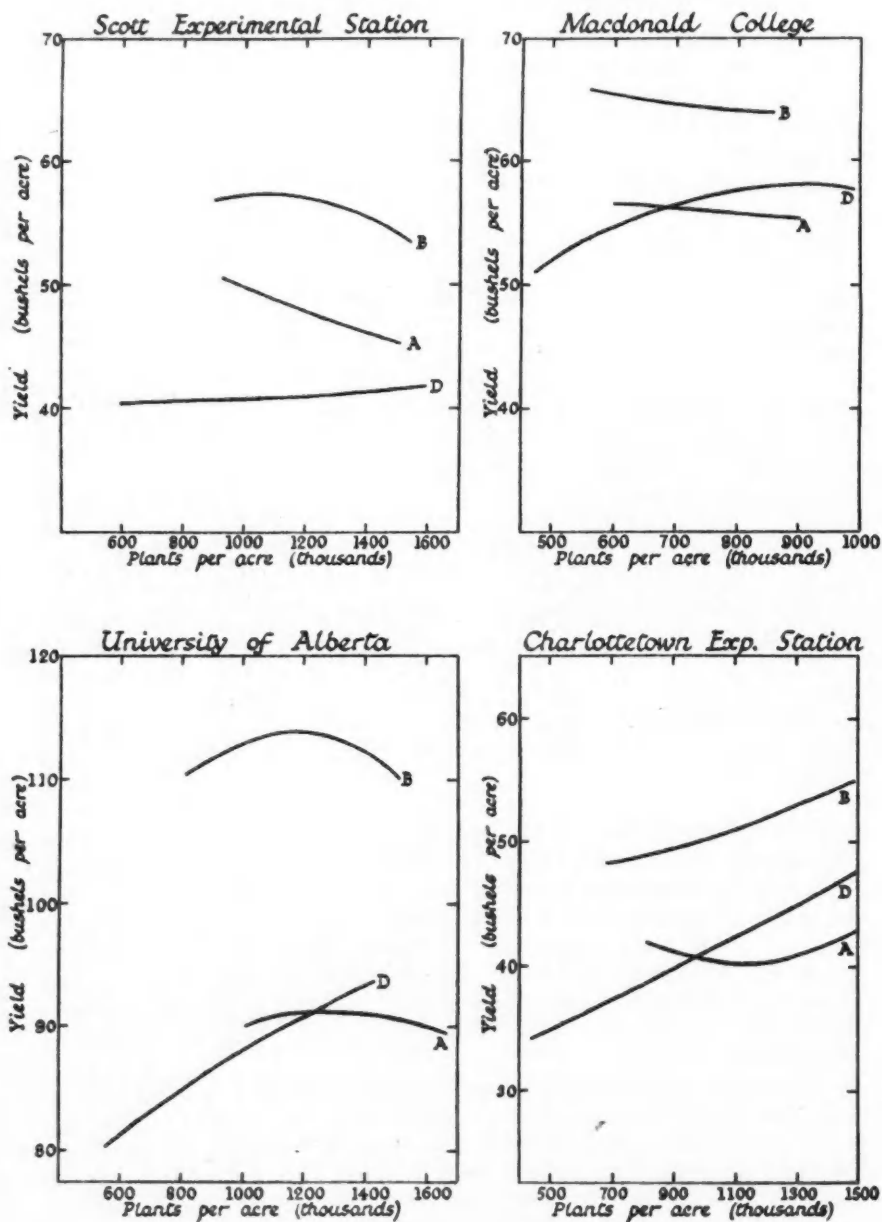


FIG. 6. Curves of partial regression of yield on plant number.
Three years' results. A = Abundance, B = Banner, D = Daubeney.

TABLE XXVII

PARTIAL REGRESSION COEFFICIENTS OF YIELD (BUSHELS PER ACRE) ON TILLER
NUMBER (THOUSANDS PER ACRE) AND "FERTILITY FACTOR"

Station	Abundance			Banner			Daubeney		
	a	b	c	a	b	c	a	b	c
Charlottetown	.00562*	-.00064	.00043	.00718*	-.00206	-.00370	.01215	.00057	.00141
Macdonald									
College	-.00306	.00093	.00241	.00477	-.00177	-.00081	.00969*	-.00220*	.00396
Scott	.00016	-.00217*	.00144*	-.00332	-.00282*	-.00019	.00296	-.00019	.00093
University									
of Alberta	-.00478	-.00090	.00170	-.00259	-.00267	-.00201	.01520*	-.00041	.00259

The number of significant values is now eight. It will be observed that the linear coefficient a appropriate to Daubeney at Charlottetown has now fallen below, whilst the corresponding coefficients of the other two varieties have exceeded the level of significance. Otherwise, the results are very similar to those of Table XXV. The value of $SS(Y-y)^2$ is however now 800.03, indicating that the relation between yield and tiller number is in general less close than that between yield and either rate of seeding or plant number. The influence of tillering upon yield would therefore appear to have been somewhat less than might have been expected. Here again however the results must be accepted with a certain reserve, and should be regarded as indications only.

Comparing $SS(Y-y)^2$ in the case of the tiller number and rate of seeding equations, we find $Z=0.1042$. This is greater than the value obtained in the case of plant number and rate of seeding, but cannot however be regarded as significant.

The values obtained for the coefficient c have been in practically all cases insignificant when compared with their standard error. It is consequently possible that the effects of soil heterogeneity have been by no means entirely eliminated from the yield comparisons. The various coefficients a and b may therefore be affected by these to an extent of which their standard errors as calculated can give no valid estimate. This possibility should be borne in mind when considering the foregoing results.

Acknowledgment

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THE EFFECT OF SUCROSE, COOKED POTATO, POTASSIUM BROMATE AND MALT UPON BAKING STRENGTH AT VARIOUS YEAST CONCENTRATIONS¹

By R. H. HARRIS²

Abstract

Three commercial and one experimental straight flour milled from 1931 Saskatchewan Marquis wheat were baked by a variety of methods, using various percentages of yeast.

Two basic formulas were used; No. 1 containing flour, water, salt and yeast, No. 2 containing the same ingredients plus sucrose. To these were added 40 cc. of cooked white potato extract and diastatic malt.

Using No. 2, no yeast starvation was apparent, but with No. 1 three flours showed this effect with higher yeast concentrations. The addition of potato extract resulted in pronounced yeast starvation with No. 1, and some starvation with No. 2. The further addition of malt produced optimum results with a yeast concentration of 3%.

A further series of bakings with another commercial flour (E) showed the improving effect of sucrose and malt in the presence of 20% of cooked white potato. Cooked sweet potato appeared to be able to support fermentation in the absence of added sucrose. Potassium bromate produced larger loaves with sugar and malt with high yeast concentrations.

As far as the flours used were concerned, using the basic formulas, there appeared to be no yeast starvation when 2.5% of sucrose was added (No. 2). When yeast stimulants were present, added sucrose and malt appeared to be necessary to prevent yeast starvation with a yeast concentration of 3%.

Introduction

A method for determining whether a baking procedure effected a separation of gassing power and flour strength has been described by Jorgensen (4). By this method, the author showed that only one of three baking procedures examined enabled an adequate distinction to be made between these flour properties. The failure of the two other test methods was due to a deficiency occurring in the supply of fermentable sugar before the loaf reached the oven, with consequent detrimental influence upon the loaf volume. Jorgensen (4) advocates a reduction in the 3% yeast content of the standard A. A. C. C. baking formula in order to offset the shortage of fermentable sugar which may arise in flours of relatively low diastatic activity. Jorgensen found the following formula quite satisfactory for the purpose of separating baking strength from diastatic activity:—yeast, 0.7%; salt, 1.25%; diastatic malt extract, 0.15%; water as per absorption; temperature of dough, 85°F. The dough time was approximately four hours, including the proofing period. The low yeast content of this formula would make a relatively small demand upon the fermentable sugar present in the fermenting dough.

Blish and Sandstedt (1), in a discussion of the differential baking test, recommended the use of 5% of sucrose in the formula when baking experimentally milled flours, to compensate for the lower diastatic activity of such

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flours and to ensure a plentiful supply of fermentable carbohydrate during the fermentation period.

Harris (2, 3) investigated the effect of sucrose upon a series of doughs, employing baking formulas which contained in one instance 3% of yeast, 2% of salt and 2.5% of sucrose. A second formula contained no sucrose but was otherwise identical with the above. The fermentation times and temperatures were in accordance with the standard method. During the course of this study it was found that, in the absence of sucrose, several of the flours produced very poor loaves, evidently as a result of yeast starvation. On the addition of an extract of cooked white potato to the doughs, the difference between the results obtained with and without added sugar was accentuated, probably as a result of yeast stimulation by the potato material without a compensating addition of fermentable carbohydrate. In the presence of sugar, satisfactory loaves were obtained, and the further addition of diastatic malt produced the largest loaves of the series. It was apparent that marked yeast starvation occurred in the absence of sucrose; it probably also occurred to a limited extent when potato extract and sucrose were used. A baking with 20% of cooked sweet potato added to the dough indicated that in the absence of sucrose this vegetable could apparently supply fermentable carbohydrate, in addition to acting as a stimulant to fermentation.

Potassium bromate in the absence of sucrose appeared to improve the loaf volume, except in the case of the two flours which showed the poorest response to the treatments with sucrose. In these yeast starvation evidently occurred. However, malt, in addition to potassium bromate and sucrose, increased the loaf volumes of these two flours, as well as the volumes of the higher protein flours.

In view, then, of the likelihood of yeast starvation occurring with higher concentrations of yeast, it seemed advisable to ascertain the effect of varying the quantity of this ingredient both above and below the standard 3% content. It was thought that this plan might give further information as to the role played by cooked potato extract in the fermentation process. Accordingly a series of bakings was carried out on several flours of different characters and baking strengths. The yeast concentrations used ranged from 1.5 to 7.5% (when sufficient flour was available). As a supplementary program, a study was made on a baker's patent, using cooked mashed potato, cooked sweet potato and malt and potassium bromate, to ascertain the effect of the various yeast concentrations upon the doughs containing these substances in addition to the regular baking formula.

Materials and Methods

The flours used in this investigation were all milled from Saskatchewan wheat of the 1931 crop. One sample, flour A, was experimentally milled from 2° Marquis containing 10.7% of protein. Flour B was a commercially milled long patent from northern Saskatchewan wheat, while flour C was also a commercial flour produced in the central portion of the province. Flour D

was a strong break flour from a mill using Marquis wheat blends. Flour E, which was used in the further tests, was quite similar to C but possessed the advantage of being available in relatively large quantities. The protein and ash contents of these flours are shown in Table I.

The baking methods used in the present study were essentially those employed by the writer in experiments on the utility of cooked potato in baking (2, 3) and will not therefore be discussed in detail here. The simple formulas Nos. 1 and 2 consisted of a basic formula: flour, salt, water and yeast in No. 1, and these ingredients plus sucrose in No. 2. The yeast content was varied in both formulas while the amounts of the other ingredients were the same. The bakings with potato, bromate, etc. were made by adding these substances to the simple formulas. The potato extract was measured

TABLE I
PROTEIN AND ASH CONTENT
OF FLOURS USED

Flour sample	Protein %	Ash %
A	9.7	0.42
B	11.7	0.60
C	13.6	0.45
D	17.1	0.46
E	13.8	0.45

by volume and added to the flour just previous to mixing. The mashed potato was weighed directly into the flour. The potassium bromate, from a stock solution containing 1 gm. in 1000 cc. of water, was added by means of a Mohr pipette. Malt, when employed, was added in the form of an aqueous solution, 4 cc. of which contained 1 gm. of diastatic malt and 3.3 cc. of water. The yeast was measured in the form of a water suspension, containing 3 gm. yeast and 7.3 cc. of water in each 10 cc. of suspension.

The potato was prepared as described previously (2, 3), by boiling sliced potatoes in a galvanized vessel. The sole difference in the preparation of the potato extract and the mashed potato consisted in allowing the water to evaporate during cooking in the latter case. It was thought desirable to determine whether or not the effects of adding mashed potato or potato extract were similar.

Results

Loaf Volumes

The loaf volumes yielded by the four flours when baked by the various methods are shown in Table II. These results are calculated to a moisture basis of 13.5%. Comparing the volumes obtained with the simple formulas, Nos. 1 and 2, there appears to be little difference in the case of flour A, except for the bakings containing 6% of yeast. Flour B shows a slight tendency toward higher volumes for formula No. 1. Flour C reveals a decided increase with formula No. 2 when 3% or more of yeast is used, and sample D behaves in a very similar manner. The average loaf volume for the four flours baked with the two simple formulas shows an increase in three cases when sucrose was included in the formula. In one flour, B, there is no evidence of yeast starvation occurring at any yeast concentration, even without added sucrose. Flours C and D tend to give lower results in the absence of sucrose when more than 3% of yeast is added, but with sucrose present in the formula the loaf volumes yielded by these samples increase with increasing yeast concentration

up to 6% of yeast. The addition of 2.5% of sucrose seems to prevent yeast starvation at any yeast concentration in any of the flours when baked with the simple formula.

TABLE II
LOAF VOLUMES OBTAINED BY THE USE OF INCREASING PERCENTAGES OF YEAST WITH
DIFFERENT BAKING FORMULAS (13.5% MOISTURE BASIS)

Yeast, %	Loaf volumes in cc.					
	Simple formula No. 1	Simple formula No. 2	40 cc. Potato extract +		Formula No. 2 +2% of malt	Average all methods
			Formula No. 1	Formula No. 2		
Flour A						
1.5	410	418	460	570	600	491
3.0	470	475	420	490	615	494
4.5	470	485	...	410	575	485
6.0	480	530	...	400	...	470
Average	457	477	440	467	597	
Flour B						
1.5	489	500	580	580	680	566
3.0	590	570	740	670	682	650
4.5	630	560	675	660	690	643
6.0	615	572	625	635	...	612
7.5	612	620	480	640	...	588
Average	587	564	620	637	684	
Flour C						
1.5	520	522	610	580	750	596
3.0	560	570	510	720	800	632
4.5	490	587	440	575	760	570
6.0	...	600	420	...	620	547
Average	523	570	495	625	732	
Flour D						
1.5	442	...	500	710	870	630
3.0	510	550	405	680	930	615
4.5	485	550	...	450	800	571
6.0	455	570	...	450	532	502
7.5	430	...	430
Average	473	557	452	544	783	
Average loaf volume for all four flours with all percentages of yeast						
	515	542	520	567	707	
Average loaf volumes for all four flours with various percentages of yeast						
Percentage yeast	1.5	3.0	4.5	6.0	7.5	
Loaf volume	571	598	567	533	509	

When the results obtained with the use of 40 cc. of potato extract are examined, a somewhat different picture is presented. Without added fermentable sugar, the loaf volume decreases with a yeast concentration greater than 1.5%, except in the case of flour B, which shows a small decrease after a yeast concentration of 3% is reached. The increased action of the yeast when stimulated by the potato material evidently accelerated the exhaustion of the available supply of fermentable carbohydrates. The addition of 2.5% of sucrose did not appear to alter materially the order of the loaf volumes, but did increase the size of the loaf throughout the series. A further addition of 2% of diastatic malt increased the loaf volumes generally, and shifted the maximum value to the 3% yeast concentration. These results would seem to indicate that with potato extract and 3% of yeast a plentiful and constant supply of fermentable carbohydrates during the fermentation period is desirable.

The average results for all the baking methods show that 3% of yeast gave the highest loaf volumes except in the case of the strong break flour, D, where only a slight decrease is noted in comparison with the value obtained with a yeast concentration of 1.5%. Apparently, then, no general yeast starvation occurred in the doughs containing 3% of yeast. These values are shown in Fig. 1. An examination of Fig. 1 shows that with all formulas, the relationship between mean loaf volume and yeast percentage is approximately linear with yeast concentrations greater than 3%, except flour B which shows the same trend beyond 4.5% of yeast. Flour A shows a poor loaf volume, due doubtless

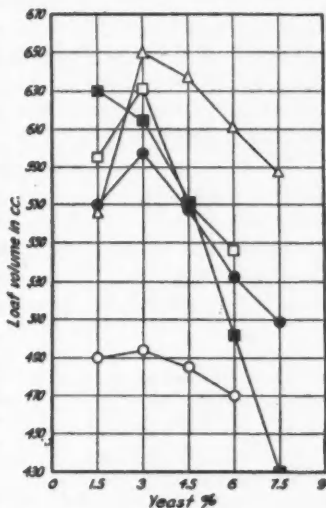


FIG. 1. Relation between mean loaf volume and yeast percentage. Legend: Mean, all flours, —; Flour A, ●; Flour B, ○; Flour C, △; Flour D, ■.

TABLE III
FORMULAS USED IN A SERIES OF EXPERIMENTS WITH FLOUR E

Series No.	Formula
1	Simple formula No. 1
2	Simple formula No. 2
3	Formula No. 1 + 20% of cooked white potato
4	Formula No. 2 + 20% of cooked white potato
5	Formula No. 2 + 20% of cooked white potato + 2% of malt
6	Formula No. 1 + 20% of cooked sweet potato
7	Formula No. 1 + .002% of potassium bromate
8	Formula No. 2 + .002% of potassium bromate + 2% of malt

to the extremely low protein content. Although sample D shows a decline after 1.5% yeast concentration, the relation does not approach a straight line function until the 3% point is reached.

The results discussed above agree with those obtained by the author (3) in that the addition of 40 cc. of potato extract accentuated the poor results obtained with those flours which yielded low loaf volumes in the absence of sucrose. It was shown also that sucrose and malt together with potato extract gave the greatest loaf volumes.

Baking Scores

A baking score was computed for the loaves baked from the flours, in the following manner:

Loaf volume.....	× 0.1		
Grain of loaf.....	× 1.0	Maximum value	10
Texture of loaf.....	× 1.0	Maximum value	10
Color of crumb.....	× 1.0	Maximum value	20

TABLE IV

LOAF VOLUMES AND BAKING SCORES YIELDED BY THE VARIOUS BAKINGS WITH FLOUR A

Yeast %	Loaf volume cc.	Color	Grain	Texture	Score
Simple formula No. 1.					
1.5	410	9.0	5	5	60
3.0	470	9.5	4	4	64
4.5	470	9.0	3	3	62
6.0	480	8.5	3	3	62
Simple formula No. 2.					
1.5	418	9.0	6.0	5.0	62
3.0	475	9.5	5.5	5.0	67
4.5	485	9.0	5.5	4.5	67
6.0	530	10.0	5.5	5.0	73
40 cc. Potato extract, simple formula No. 1					
1.5	460	12	5	5	68
3.0	420	10	4	4	60
40 cc. Potato extract, simple formula No. 2					
1.5	570	12.0	5.0	5	79
3.0	490	13.0	6.0	6	74
4.5	410	10.0	5.5	5	61
6.0	400	10.5	5.0	5	60
40 cc. Potato extract, simple formula No. 2, 2% of malt					
1.5	600	10.0	5.0	4	79
3.0	615	10.0	4.0	4	79
4.5	575	10.5	4.5	4	76

The color of the crumb was given a heavier weighting than the other interior characteristics on account of the popular demand for a white loaf. Symmetry of loaf was not considered in the present study.

In Tables IV, V, VI and VII the detailed and final baking scores for the four flours are shown. Table VIII summarizes the scores according to the percentage of yeast used in the baking, and Table IX summarizes Table VIII. These tables show a general tendency toward a decline in the score with a yeast concentration over 3%, and further strengthen the conclusions derived from a consideration of the loaf volumes alone. In other words, the optimum results were obtained with a yeast concentration of 3% in the entire series of bakings, with the exception of flour D which showed a slightly higher value with a yeast concentration of 1.5%.

The unbleached samples A and D reveal a slightly higher color score on the addition of potato extract.

Fig. 2 shows graphically the effect of increasing yeast concentration on the final baking score. A similarity is evident between the relationships depicted in this figure and that of loaf volume-yeast concentration shown in Fig. 1, the same tendency toward a straight line being shown with yeast concentrations greater than 3%. Flour A is very low, while sample B has the highest score throughout the yeast range.

The Effect of Cooked Potato

In order to determine the effect of cooked mashed potato in conjunction with various concentrations of yeast, further series of bakings were undertaken, using flour E. In each series the yeast concentration varied from 1.5 to 7.5%. The formulas used for these series are shown in Table III, and the results obtained on baking are shown in Table X.

It will be seen from an examination of Table X that series No. 1 showed a maximum loaf volume with a 3% yeast concentration, while in series No. 2, where sugar was added, the loaf volume increased up to a yeast concentration of 6%. This difference between the two series was of course due to yeast starvation in series No. 1.

The effect of adding cooked white potato is shown by the results of series No. 3 and 4. The loaf volume in both cases is greater than that from the analogous series (No. 1 or No. 2), with the exception of the 7.5% yeast value for series No. 3. This effect is not so marked in series No. 3, where the maximum loaf volume is obtained with a yeast concentration of 3%, as in series

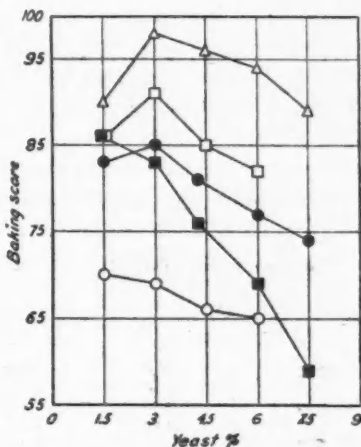


FIG. 2. Relation between mean baking score and yeast percentage. Legend: As in Fig. 1.

No. 4, where the largest loaf is obtained with 4.5% of yeast. This was no doubt due to yeast starvation brought about by rapid exhaustion of the available sugar, which exhaustion was in turn due to stimulation of the yeast by the potato.

Support for this hypothesis is given by the results of series No. 5, in which the plentiful supply of fermentable carbohydrate makes it possible to use a yeast concentration as high as 6%, and produces the largest loaf of all the bakings. The flour was apparently strong enough to stand the increased gas production.

TABLE V

LOAF VOLUMES AND BAKING SCORES YIELDED BY THE VARIOUS BAKINGS WITH FLOUR B

Yeast %	Loaf volume cc.	Color	Grain	Texture	Score
Simple formula No. 1					
1.5	489	20.5	8	8	85
3.0	590	21	8	8	96
4.5	630	21	6	6	96
6.0	615	21	4	5	91
7.5	612	19.5	5	5.5	91
Simple formula No. 2					
1.5	500	21.5	7	7.5	86
3.0	570	21	7	7.5	92
4.5	560	21	7	7.5	91
6.0	572	20.5	8	8	94
7.5	620	21	5	6	94
40 cc. potato extract, simple formula No. 1					
1.5	580	21	6	6.5	91
3.0	740	21	4	5	104
4.5	675	21	5	5.5	99
6.0	625	20	6	6	94
7.5	480	19	4	5.5	76
40 cc. potato extract, simple formula No. 2					
1.5	580	22	7	7	94
3.0	670	22	5	6	100
4.5	660	21	6	6	99
6.0	635	20.5	6.5	6.5	97
7.5	640	20	6.5	6.5	97
40 cc. potato extract, simple formula No. 2, 2% malt					
1.5	680	18	5	4	95
3.0	682	18.5	5.5	4	96
4.5	690	19	4	3	95

The effect of the addition of cooked sweet potato is shown in the results of series No. 6. Inasmuch as not only are the loaves larger throughout than in series No. 1, but also the loaf volume increases up to a yeast concentration of 4.5%, it is apparent that this material not only stimulates fermentation but also supplies fermentable carbohydrate.

The results of series No. 7 and 8 also show larger loaves than series No. 1 and 2, with maximum volumes at a yeast concentration of 4.5% of yeast.

Throughout all eight series, poor results were obtained with a yeast concentration of 7.5%, again due no doubt to yeast starvation which even malt and sweet potato could not prevent.

The use of 4.5% of yeast produced the largest loaves in five of the series (Nos. 2, 4, 6, 7, 8) while a concentration of 3%, the quantity prescribed by

TABLE VI

LOAF VOLUMES AND BAKING SCORES YIELDED BY THE VARIOUS BAKINGS WITH FLOUR C

Yeast %	Loaf volume cc.	Color	Grain	Texture	Score
Simple formula No. 1					
1.5	520	16	6	7	81
3.0	560	17	6	7	86
4.5	490	15	5	5.5	79
Simple formula No. 2.					
1.5	522	16	6	5	79
3.0	570	17	6	6	86
4.5	587	17.5	7	6	89
6.0	600	17.5	7	6.5	91
40 cc. potato extract, simple formula No. 1					
1.5	610	19	4	4.5	88
3.0	510	17	5	5	78
4.5	440	16	4.5	5	69
6.0	420	15.5	4.5	4	66
40 cc. potato extract, simple formula No. 2					
1.5	580	18.5	4	5	85
3.0	720	19.5	3	4	98
4.5	575	18	5	5	85
40 cc. potato extract, simple formula No. 2, 2% of malt					
1.5	750	16	4	4	99
3.0	800	17	4.5	4.5	106
4.5	760	18	4.5	4	102
6.0	620	17.5	5	5	89

TABLE VII

LOAF VOLUMES AND BAKING SCORES YIELDED BY THE VARIOUS BAKINGS WITH FLOUR D

Yeast %	Loaf volume cc.	Color	Grain	Texture	Score
Simple formula No. 1					
1.5	442	9.0	7.5	6	67
3.0	507	11.5	7.0	7	76
4.5	485	9.0	6.0	7	70
6.0	455	9.0	6.0	7	67
Simple formula No. 2					
3.0	550	11.0	7.0	7.0	80
4.5	550	12.0	7.0	7.0	81
6.0	570	12.5	7.5	7.5	84
40 cc. potato extract, simple formula No. 1					
1.5	500	10.5	5	5	70
3.0	435	10.0	3	4	60
40 cc. potato extract, simple formula No. 2					
1.5	710	15.0	7	7.0	100
3.0	680	12.0	6	6.0	92
4.5	450	10.0	4	3.0	62
6.0	450	9.5	4	3.5	62
7.5	430	9.5	4	3.0	59
40 cc. potato extract, simple formula No. 2, 2% of malt					
1.5	870	12.0	3.0	4	106
3.0	930	11.0	1.5	3	108
4.5	800	10.5	1.0	2	93
6.0	532	10.5	0.5	1	65

TABLE VIII

SUMMARY OF SCORES ACCORDING TO YEAST PERCENTAGE

Yeast %	Color	Grain	Texture	Final score	Yeast %	Color	Grain	Texture	Final score
Flour A					Flour C				
1.5	10.4	5.2	4.8	70	1.5	17.1	4.8	5.1	86
3.0	10.4	4.9	4.6	69	3.0	17.5	4.9	5.3	91
4.5	9.6	4.7	4.1	66	4.5	16.9	5.2	5.1	85
6.0	9.7	4.5	4.3	65	6.0	16.8	5.7	5.1	82
Flour B					Flour D				
1.5	20.6	6.6	6.6	90	1.5	14.9	5.5	5.5	86
3.0	20.7	5.9	6.1	98	3.0	14.9	5.1	5.3	83
4.5	20.6	5.6	5.6	96	4.5	14.4	5.0	4.9	76
6.0	20.5	6.1	6.4	94	6.0	14.3	5.6	5.1	69
7.5	19.9	5.1	4.7	89	7.5	14.7	4.5	3.8	59

the official A. A. C. C. method, produced the largest loaf in two series only, this being due to the low fermentable carbohydrate content of these series.

It is of interest to note that series No. 7 and 8 produced comparatively large loaves with high yeast concentrations.

These results are in agreement with those previously obtained by the author (2, 3).

The substitution of 20% of white cooked potato for the 40 cc. of white potato extract appeared to give strong stimulation. This effect was most marked in the absence of sucrose.

TABLE IX

AVERAGE SCORES OF ALL BAKINGS WITH THE FOUR FLOURS ARRANGED ACCORDING TO INCREASING CONCENTRATION OF YEAST

Yeast c/c	Color	Grain	Texture	Final score
1.5	14.9	5.5	5.5	83
3.0	14.9	5.1	5.3	85
4.5	14.4	5.0	4.9	81
6.0	14.3	5.6	5.1	77
7.5	14.7	4.5	3.8	74

TABLE XII

MEAN LOAF VOLUMES AND BAKING SCORES FOR THE VARIOUS YEAST CONCENTRATIONS USED IN SERIES 1 TO 8 WITH FLOUR E

Yeast c/c	Mean loaf volume cc.	Mean baking score
1.5	592	88
3.0	662	92
4.5	681	94
6.0	642	88
7.5	583	80

The baking scores assigned the loaves in these series of bakings are, with the exception of series No. 6, shown in Table XI. Series No. 6 has been omitted because of the exceedingly low color, grain and texture score. Considerable variation in crumb color is evident among the loaves produced by the different methods. The addition of sucrose and cooked potato (Series No. 4), or sucrose, malt and cooked potato (Series No. 5) appeared to reduce the color score. The bromate bakings without malt (series No. 7) yielded the highest color score for the series, but the grain and texture were poor. The inclusion of sucrose (series No. 2, 4,) and sucrose and malt (Series No. 5, 8) caused the maximum baking score to fall at a yeast concentration higher than 3%, the

TABLE X

LOAF VOLUMES OBTAINED IN THE BAKINGS OF SERIES 1 TO 8 WITH FLOUR E

Yeast %	Series No.*							
	1	2	3	4	5	6	7	8
1.5	530	522	600	640	665	565	600	615
3.0	600	572	750	720	720	640	670	625
4.5	560	587	725	775	750	695	690	670
6.0	470	586	580	762	810	645	620	660
7.5	440	550	500	680	695	605	570	630

* See Table III. Simple formula No. 1 includes flour, water, salt and yeast, simple formula No. 2 includes flour, water, salt, yeast and sucrose.

TABLE XI

BAKING SCORES ASSIGNED THE LOAVES BAKED IN SERIES NO. 1 TO 8, WITH FLOUR E

Yeast %	Loaf volume cc.	Color	Grain	Texture	Score
Series No. 1					
1.5	530	17.0	7.5	8	85
3.0	600	18.0	7	6	91
4.5	560	17.5	7.5	7	88
6.0	470	16.0	5	5	73
7.5	441	15.0	5	3	67
Series No. 2					
1.5	522	17.0	7	7.0	83
3.0	572	17.5	7	7.5	89
4.5	587	18.0	7	8.0	92
6.0	586	17.0	6	8.0	90
7.5	550	17.5	6	6.0	84
Series No. 3					
1.5	600	16	8	8	92
3.0	750	18	6	6	105
4.5	725	19	5	6	102
6.0	580	15	6	7	86
7.5	500	14	5	7	76
Series No. 4					
1.5	640	12	7	8.0	91
3.0	720	15	5	7.0	89
4.5	775	14	4	6.0	101
6.0	762	13	4	5.5	98
7.5	680	12	4	5.5	89
Series No. 5					
1.5	665	11	3	3	83
3.0	720	10	3	2	87
4.5	750	10	4	3	92
6.0	810	10	3	3	97
7.5	695	10	4	4	87
Series No. 7					
1.5	600	19	8.5	9.0	96
3.0	670	20	9.0	8.0	104
4.5	690	20	6.0	6.0	101
6.0	620	18	5.0	6.0	91
7.5	570	17	5.0	6.5	85
Series No. 8					
1.5	615	12.0	5.5	5	84
3.0	625	11.0	5.0	5	83
4.5	670	13.0	3.0	3	86
6.0	660	10.0	2.0	3	81
7.5	630	9.5	1.0	1	74

optimum for the bakings without these ingredients (Series No. 1, 3, 7). The mean loaf volume and score for all the methods show the optimum yeast concentration to be 4.5%. These values are shown in Table XII.

General Summary and Conclusions

1. There was no apparent yeast starvation when a series of four flours were baked with the addition of 2.5% of sucrose at any yeast concentration used in this study. Without sucrose, detrimental effects upon the loaf were evident at the higher yeast concentrations for three of the flours.

2. Potato extract caused yeast starvation at higher yeast contents in the absence of added sugar. This effect persisted to a lesser degree in the presence of 2.5% sucrose. The further addition of 2% diastatic malt obviated yeast starvation with 3% of yeast for all the flours. The highest mean loaf volume and baking score of any method was given by such a formula. From these results, sucrose and diastatic malt in the concentrations used would appear to be necessary if yeast starvation is to be avoided with a yeast concentration of 3%.

3. Cooked white potato served to support fermentation somewhat better than potato extract in the absence of added sugar. Sucrose, or sucrose and malt, however, effected a progressive increase in loaf volume and shifted the maximum loaf volume to a higher yeast concentration.

4. Cooked sweet potato appeared to supply fermentable carbohydrate as well as exerting a stimulating effect upon fermentation, though this stimulating effect would seem to be less than the somewhat similar action of cooked white potato.

5. Potassium bromate yielded very similar results with and without sucrose and malt with the exception that somewhat better results were obtained with very high yeast concentrations.

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REPLACEABLE BASES, HYDROGEN AND BASE-HOLDING CAPACITY OF ALBERTA SOILS¹

BY N. HOLOWAYCHUK²

Abstract

Studies on soils from three major soil groups in Alberta showed relatively high content of replaceable bases with normal calcium, magnesium and sodium-potassium ratios. The A_2 horizon of the wooded soils was lowest in replaceable bases and indicates excessive leaching. Excessive leaching has not occurred in any of the horizons of the brown and the black soils. Losses by leaching in the brown and black soils have resulted mainly from the movement of water-soluble cations.

Movement of base-exchange complexes from the A_2 horizon of the wooded soils was apparently due to dispersion rather than disintegration. The excess SiO_2 in the A_2 horizon has resulted from disintegration of feldspars rather than from the breaking up of the base-exchange complex.

Greater proportions of hydrolytic acidity were found in the A_2 and B_1 horizons of the wooded soils than in the black soils.

The wooded soils appear to belong to the podsol group according to the Gedroiz system of classification.

Introduction

The question of base-exchange in soils has been given considerable attention, especially in the last decade, and a good deal of work has been done at various institutions on this continent and in Europe. It is the intention of the writer to show in this report how the soils of Alberta compare with each other and with those from other parts of the world.

Way was the first to report on base-exchange in soils, and his findings are well summarized by Hissink (4). Following this, little work was done, or at least made available, until about 1920 when the works of Hissink and Gedroiz were translated. Since then an ever-increasing number of workers have taken part in these investigations, resulting in an extensive literature.

The study of base-exchange in soils as developed by present-day workers seems to resolve itself into the following four lines of activity:

(1) The study of the chemistry of ion exchange in soils. (2) Isolation and identification of the soil fractions having base-exchange properties. (3) The study of the effect of the dynamic forces of the soil on base-exchange properties of the soil. (4) The study of the agronomic significance of base-exchange in soils.

A striking feature of base-exchange reaction is the rapidity with which the equilibrium is reached. These are apparently surface reactions in which all the exchangeable ions are easily accessible. Hissink and Gedroiz (3, 4) explain base-exchange as physico-chemical rather than non-polar absorption or the double decomposition of an insoluble salt. These however may occur to some extent. Both Gedroiz and Hissink looked upon the exchange complex as being an insoluble anion (a colloidal particle) with an electrical double layer where the cations are held, partially adsorbed on the surface and

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partially as an atmosphere of dissociated ions. This concept is generally held with respect to base-exchange, and the base-exchange properties of the soil are attributed to the colloidal fraction, organic or mineral. The properties of the soil, such as dispersion, flocculation, swelling, structure, etc., can be explained by observing the effect of the exchangeable cation of the electrical double layer. It is found that cations of low molecular weight and valency are more hydrated and have a higher osmotic pressure, thus increasing the dispersion, swelling, etc., of the soil. These effects are decreased as the cation increases in valency and molecular weight.

The isolation and identification of the complex showing base-exchange properties have been given considerable attention by the workers in this field. Way was the first to notice that the seat of base-exchange reactions was in the clay fraction of the soil. Gedroiz and Hissink have attributed this property to certain alumino-silicates in the clay fraction, and have also found that the "humus" or "humic" portion of the soil is very important in this respect. They found also a very striking similarity between this exchange complex in the soil and ordinary zeolites, and this fact has led many workers to adopt the view that base-exchange in the soil was due, in part at least, to natural zeolites. Kerr (8) more recently was able to extract from the soil a mineral fraction which was mainly responsible for base-exchange in the mineral portion. He compared this fraction with zeolites as to exchange equilibria, stability and specific gravity and found no correlation. Analysis of this fraction by Truog and Chucka (15) on further purification, showed that it is an alumino-silicate with an $\text{Al}_2\text{O}_3 : \text{SiO}_2$ ratio of about 1:4. Kelley, Dore and Brown (7) further investigated this point by subjecting natural zeolites, bentonites and soil colloids to X-ray and stability studies. They found no resemblance in the X-ray spectra of zeolites and soil colloids but a resemblance was noticed in the bentonites and soil colloids. Also, the soil colloids and bentonites were more or less alike, in regard to stability, when subjected to heating, both being much more stable than zeolites. Presumably the base-exchange complex is an hydrated alumino-silicate, rather complicated in structure, and is one of the secondary stages of weathering of rocks.

High base-exchange capacity of organic soils has led some workers to investigate this field. McGeorge (10) found a high correlation between the lignin fraction and base-exchange capacity. There was less correlation when cellulose and hemi-cellulose were compared.

Important dynamic forces such as the climate and water movements act on the soil. Thus, where the climate is cool and moist considerable organic matter is produced having a wider carbon-nitrogen ratio than is found in the more arid regions. Decomposition under a cool moist climate is slow and likely to produce acid end-products. The soil solution in this case tends to become more acid than it would under drier and warmer conditions. Thus, the reaction of the soil solution as well as its movements and quantity are important in determining the development of the soil, and as the cations exert a marked influence on the physical properties of the soil colloids, the effect of the soil solution on these cations should be considered.

Gedroiz (3) noticed that if a soil was leached with water the basic cations were gradually removed and gave rise to an acid complex. He found that this was somewhat more dispersed than where calcium was the main complex and tended to move with the general water movements of the soil. He also found that if this complex was sufficiently acid it would disintegrate into its oxides—mainly aluminium oxide and silicon dioxide. Thus the hydrolytic action of pure water tends to produce what is essentially a process of podsolization, and this action is increased if the soil is made acid by decomposing organic matter as mentioned previously.

Where the alkali cations predominate, *e.g.*, sodium, the colloids are in a dispersed state. As the sodium complex hydrolyzes easily when not prevented by the common-ion effect of sodium salts, the resulting sodium hydroxide is leached away and an acid complex (soloti) remains.

Gedroiz has actually worked out a system of soil classification based on the degree of saturation and also on the basic cation content of the soil. This system is given by Afanasiev (1) and contains two large groups, *vis.*, podzols and laterites where the complex is unsaturated with basic cations, and chernozems and alkaline where it is saturated.

Gedroiz and Hissink were the first to develop the idea of unsaturation. The exchange complex is said to be unsaturated when some of the exchangeable cations are hydrogen. The degree of saturation is the total quantity of basic cations compared to the total capacity of the complex for cations.

Base-exchange has a significant role in agriculture. Soil reaction, soil structure and availability of cations are more or less directly connected with base-exchange. In considering soil structure Hissink (4) found that soils with 35% of replaceable, univalent cations had poor tilth but were satisfactory if these cations were reduced to 20%. Somewhat similar results are produced by unsaturation. Turner (16) working in Trinidad has estimated that poor conditions are produced where the unsaturation is about 40%. Soil acidity is also directly affected by base-exchange. In this respect, the degree to which the complex ionizes and the ratio of hydrogen to basic ions determine the active acidity. The question of unsaturation and availability of cations has been dealt with by Robinson and Williams (14). They found that available calcium varies directly as the exchangeable calcium and unsaturation of the soil. Where exchangeable calcium was high, the question of unsaturation was less vital. Pierre (13) was able to obtain a fair correlation between plant injury and soil unsaturation. It may be possible that the toxicity of the more acid soils is due to certain elements, for example, Kelley and Brown (6) found replaceable aluminium, manganese and iron in some acid soils. Fertilization of soils is also influenced by base-exchange. Kelley (5) has discussed cases in which some difficulty was experienced in getting potassium down to deep-rooted plants. In these cases potassium was fixed in the surface horizons. The ammonium radical is similarly fixed when ammonium salts are applied. In all these cases the predominant exchangeable cation must be taken into account. The more fertile soils are high in replaceable bases with calcium as the dominant cation.

The data and discussion given in this paper have been planned to compare the Alberta soil groups in regard to the vertical distribution of their replaceable bases, base-holding capacity, and replaceable and hydrolytic acidity. Comparisons are also made with soils reported from Illinois and Rothamsted.

Description of Alberta Soils

The selection of samples used in this investigation has been such that they represent the three major soil groups found in Alberta. These soils are grouped according to the kind and degree of weathering to which they have been subjected. The extent and analysis of these groups have been reported on by Wyatt and Newton (17) and Leahey (9).

In the south of the province there are the brown soils, which are the result of weathering under low precipitation and high evaporation. Such conditions are not conducive to excessive growth and accumulation of organic matter, hence the quantities of organic acids produced will not be very great. It may be seen also that with low precipitation and high evaporation there will be very little water passing downwards. Most of it penetrates to a shallow depth and returns to the surface by evaporation and transpiration. The absence of percolation accounts for accumulation of salts near the surface. Under these conditions there will be little or no movement of soil colloids. The exchangeable complex present in these soils is mainly mineral and very little organic matter is present.

The next distinct soil group consists of the black soils. These are produced under moister conditions than were the brown. Here the evaporation is less, but the downward movement of water is still not excessive. This climate is conducive to active growth and grass is usually the main covering. Thus it may be seen that only the more soluble parts will be moved. There has been a movement of electrolytes, such as calcium carbonate, but this has not been excessive, and as a rule there is sufficient to hold the soil colloids.

The third group consists of wooded soils, composing about two thirds of the area of the province. They are the least fertile of the groups and are produced under more humid conditions than either of the other two. These humid conditions may be brought about by higher precipitation, lower evaporation, poor drainage or a combination of these. The vegetation in this area is fairly rank, consisting mainly of trees, shrubs, moss and some grass. The soil is maintained in a humid condition, thus providing water for the leaching process. This water is slightly acidic, mainly from carbon dioxide produced by the decay of organic matter. The leaching power of the water is increased, and thus there is a downward movement of the bases released by the acid conditions. The removal of bases makes the soil colloids less stable. Thus there is a horizon in the upper part of the soil where a removal of bases and colloids has occurred, with subsequent precipitation farther down if the soil has been weathered long enough.

The precipitation in Alberta is usually between 10 and 20 in., hence the differences in soils have been brought about mainly by differences in evaporation.

The movement of bases and colloidal fractions in the above soils should be indicated by the content of replaceable bases and the base-holding capacity of the various horizons.

In general the soil profile is characterized by three general horizons, designated *A*, *B* and *C*. The surface horizon *A* has undergone some weathering. Below this there is horizon *B* showing an accumulation of degradation products from *A*. The more or less unaltered horizon underlying *A* and *B* is designated by *C*.

This general grouping of soil horizons is frequently subdivided further, as follows: *A*₀, the layer of raw or partially decayed organic matter; *A*₁, the horizon of accumulation of decayed products from *A*₀, if these are not leached away; *A*₂, the horizon where the leaching has occurred to greatest extent; *A*₃, the horizon of transition between *A*₂ and *B*₁; *B*₁, the horizon of accumulation of colloidal matter from *A* horizons; *B*₂, the horizon of precipitation of electrolytes from above, mostly as carbonates; *C*, the horizon which has not undergone any appreciable weathering.

Five profiles were taken from the wooded area, three from the black soils and two from the brown. Those from the wooded area were taken at Fort Vermilion, Ksituan (near Spirit River), Breton, Wabamun and Alberta Beach, those from the black soil area were from Edmonton and Hobbema, and the two brown profiles were taken at Shouldice and Benton.

Experimental

The methods of extraction employed in base-exchange work may be classified into three groups: (1) Treatment of the soil with a neutral salt; (2) Treatment of the soil with a dilute acid; (3) Electrodialysis.

Ammonium acetate was used as the extracting solution and so the method belongs to the first group. There are two distinct advantages in using ammonium acetate as the extracting solution: (1) ammonium acetate is a neutral salt and has pronounced buffer properties around pH 7; (2) it is easily expelled by a single ignition.

However a very considerable amount of work has been done using ammonium chloride, so it was thought advisable to compare these two salts. A series of samples was run using each solution and the replaceable calcium and magnesium determined. The results, shown in Table I, were consistent throughout, and no important difference was noticed between the two salts.

The procedure of extracting a soil sample with the salt solution and the subsequent determination of the bases replaced were as follows: the air-dry soil was mixed and a portion of it crushed to pass a 20-mesh sieve. Twenty-five grams of the sifted soil was weighed out and digested overnight with 500 cc. of *N* ammonium acetate (pH 7). The sample was filtered the following day on a Buchner using suction, and then leached with more ammonium acetate till a litre of the solution was obtained.

A 100-cc. sample (or larger if the base contents were low) of the solution was evaporated on a steam bath. A little nitric acid was added and, when dry, the sample was ignited over a Meker burner. The residue was dissolved in hot water with

the addition of 2-3 cc. of hydrochloric acid, and ammonia separation carried out to remove aluminium and iron if present. Calcium was precipitated in the filtrate from the latter as oxalate and titrated with 0.1 *N* potassium permanganate. The filtrate from the calcium determination was evaporated and ignited to remove all ammonium salts. Following this the residue was evaporated two or three times with sulphuric acid, ignited at about 700°C., and then the sulphate held by bases determined as barium sulphate. The sulphate expressed in equivalents represents mostly the magnesium and sodium-potassium that have been replaced*. Magnesium was determined in a separate quantity of solution by Epperson's method (Treadwell and Hall) after aluminium, iron and calcium were removed as indicated above. The base-holding capacity was determined by leaching 25 gm. of air-dry soil with a litre of *N* calcium chloride on a Buchner. Following this it was washed free of chlorides and then the calcium fixed was replaced and determined as in the regular method.

Hydrolytic acidity was determined by shaking 25 gm. of soil with 250 cc. of normal calcium acetate for one hour, filtering and titrating with 0.1 *N* potassium hydroxide. Phenolphthalein was used as indicator.

Replaceable acidity was determined by shaking 25 gm. of soil with 125 cc. of normal potassium chloride for one hour, filtering and titrating with 0.1 *N* potassium hydroxide using phenolphthalein as indicator. This method was checked by using 250 cc. of normal barium chloride in place of potassium chloride. The results obtained are shown in Table II and did not show any pronounced differences.

There is a marked removal of replaceable bases in the *A*₂ horizon of the wooded profiles. Of these, the Alberta Beach profiles show the least leaching, but this does not represent the most degraded of the author's profiles. The *A*₁ horizon of this profile has the least replaceable bases, but it has fair structure

TABLE I
REPLACEABLE CALCIUM AND
MAGNESIUM OBTAINED BY AMMONIUM
CHLORIDE AND AMMONIUM ACETATE
METHODS*

Calcium		Magnesium	
NH ₄ Ac	NH ₄ Cl	NH ₄ Ac	NH ₄ Cl
<i>Bretton B₁</i>			
18.8	19.2	6.7	9.0
18.8	19.0	7.0	8.8
17.6	18.5	6.5	8.7
17.6	18.6	6.3	9.2
<i>Bretton A₂</i>			
7.5	7.1	1.5	1.5
7.6	6.8	1.4	1.7
7.6	6.8	1.5	1.5
7.7	—	1.5	—
<i>Bretton A₁ and A₂</i>			
25.5	23.0	4.6	4.9
25.2	23.9	4.8	5.1
24.5	22.5	4.9	4.7
24.1	22.8	4.9	4.7
<i>Edmonton A₂</i>			
24.7	28.3	8.4	8.9
25.2	28.5	9.0	8.7
<i>Edmonton B₁</i>			
18.8	18.0	9.5	7.1
18.8	19.4	8.8	7.5
17.1	20.1	9.3	6.9
22.6	19.6	9.4	6.5

*Results expressed as milli-equivalents per 100 gm. of soil.

*This method is not very satisfactory for the determination of small quantities of sodium-potassium. However, it gives a fair indication when appreciable quantities are present.

and some organic matter incorporated in it. The A_2 horizon in this profile also lacks the extremely bleached color found in the corresponding horizon of the other wooded profiles.

TABLE II
REPLACEABLE BASES, HYDROGEN AND BASE-HOLDING CAPACITY OF ALBERTA SOILS

Horizon	Depth, in.	Calcium	Magnesium	Total bases	Sodium, potassium etc.	Base holding capacity	Hydrogen		Ratio,†	Ratio,‡	Ratio,§
							Replaceable	Hydrolytic			
Breton (wooded)											
A ₀ , A ₁	0-2	24.8	4.9	29.8	0	30.4	0	6.1	99	100	83
A ₂	2-18	6.7	2.3	8.7	0	10.5	0	2.2	83	100	80
B ₁	18-42	15.1	6.5	20.3	0	28.9	1.8	5.5	70	92	78
C	At 84	41.5	5.7	46.7	0
Wabamun (wooded)											
A ₀	1	37.8	5.9	43.7	0	46.8	0	93	99
A ₁	1-3	11.2	2.8	14.0	0	12.3	0	.8	114	100	95
A ₂	3-12	8.7	3.0	12.1	0	9.2	0	1.7	132	100	88
B ₁	12-60	17.8	7.0	25.5	0	22.7	1.5	4.8	112	96	84
B ₂	At 60	72.5	7.2	79.9	0
Ft. Vermilion (wooded)											
A ₁	0-3	20.0	8.0	31.7	3.7	25.5	.2	124	99
A ₂	4-8	3.7	2.1	7.2	1.4	15.0	2.3	6.0	50	76	55
B ₁	8-18	14.8	13.8	35.8	7.0	42.2	0	1.0	83	100	97
B ₂	At 27	100.0	11.3	112.2	.9
Kaituan (wooded)											
A ₀ , A ₁	0-5	35.6	5.7	41.3	0	40.0	0	102	100
A ₂	5-12	7.5	2.7	10.2	0	9.5	.9	3.0	108	92	77
A ₃	12-22	14.1	10.2	24.5	0	28.5	4.5	7.0	87	84	77
B ₁	22-27	18.2	12.3	30.9	0	28.9	0	1.7	107	100	95
B ₂	At 30	46.3	12.0	56.3
Alberta Beach (wooded)											
A ₁	2-5	11.7	3.9	17.6	2.0	19.0	0	4.1	93	100	81
A ₂	5-12	9.9	3.5	15.2	1.9	12.3	0	1.9	123	100	89
A ₃	12-18	12.0	3.7	15.9	.2	16.3	0	1.7	98	100	90
B ₁	At 36	14.4	5.9	21.6	1.3	18.6	0	1.7	116	100	91
B ₂	At 72	56.0	5.8	63.2	1.4
C	At 72	48.3	8.0	56.7	.3
Edmonton 634 (black)											
A ₁	1-12	50.0	6.4	57.4	1.0	51.0	0	112	100
A ₂	12-24	30.5	8.9	41.4	2.0	37.0	0	1.1	110	100	98
B ₁	24-30	27.5	8.2	35.7	0	32.4	0	.7	112	100	98
B ₂	30-48	73.0	6.3	79.3	1.5
C	67.0	9.1	76.1	0.7
Hobbema (black)											
A ₁	0-12	31.0	10.0	39.9	0	45.6	.2	6.5	87	99	86
A ₂	12-16	15.9	6.7	22.2	0	25.4	.0	2.6	88	100	89
B ₁	16-30	18.4	8.7	27.3	0	29.7	.0	1.2	92	100	96
B ₂	30-36	90.7	9.1	100.2	0
C	60-72	28.5	9.4	37.2	0
Edmonton 649 (black)											
A ₁	0-5	34.0	7.1	42.2	1.0
A ₂	5-13	25.0	8.7	31.6	0
B ₁	13-27	18.0	9.2	26.6	0
B ₂	27-42	66.5	10.0	74.0	0
C	At 48	22.0	9.1	30.0	0

TABLE II—Continued

Horizon	Depth, in.	Calcium	Magnesium	Total bases	Sodium, potassium etc.	Base holding capacity	Hydrogen		Ratio, [†]	Ratio, [‡]	Ratio, [§]
							Replaceable	Hydrolytic			
Benton (brown)											
A ₁	0—5	26.5	5.4	33.7	1.8	41.8	Alk.	.3	81	100+
A ₂	5—10	20.3	5.7	28.3	2.3	22.5	Alk.	0	127	100+
B ₁	10—12	25.8	6.9	33.6	.9	25.3	Alk.	136	100+
B ₂	12—24	76.7	11.9	89.0	.4
C	At 48	48.0	12.1	62.0	1.9
Shouldice											
A ₁	0—7	58.0	6.0	64.0	0	60.5	Alk.	106	100+
A ₂	7—17	115.0	5.1	120.0	0	93.0	Alk.	140	100+
B ₁		88.5	10.5	99.0	0	95.0	Alk.	104	100+
B ₂		95.5	23.1	121.5	2.9

*Results expressed as milli-equivalents per 100 gm. of soil, $\dagger \text{Ratio}_1 = \frac{\text{total bases}}{\text{base-holding capacity}} \times 100$,

$\ddagger \text{Ratio}_2 = \frac{\text{total bases}}{\text{total bases} + \text{replaceable hydrogen}} \times 100$, $\S \text{Ratio}_3 = \frac{\text{total bases}}{\text{total bases} + \text{hydrolytic hydrogen}} \times 100$.

The most significant difference found in replaceable-base content of the different soils was in the A₂ horizon. In the better developed wooded profiles it was between 7.2 and 12.1 M.E. (milli-equivalents) per 100 gm. of soil. In the black soils this was between 22.2 M.E. (Hobbema) and 41.4 M.E. (Edmonton). There is no apparent leaching in this horizon in the brown soils and as a matter of fact, an accumulation occurs in Shouldice profile.

There are no great differences in the replaceable base contents of the B₁ horizons of any of the profiles studied, but in the wooded soils, the replaceable bases in this horizon are at least double the total replaceable bases found in the A₂ horizon.

The wooded profiles taken individually show a replaceable base content in the A₀ horizon that compares well with that of the A₁ horizon from the black soils. The A₂ horizon shows a decided removal of replaceable bases when it is compared with the other horizons of the same profile, or with the A₂ horizon of the black or brown soils.

The black-soil profiles, with the exception of the A₂ horizon, show a replaceable base content somewhat similar to the wooded profiles. The A₂ horizon of the black soils shows no pronounced removal of bases as it does in the wooded profiles.

In case of the brown soils there is no movement of replaceable bases indicated. The Benton profile does not show any great variation in the A₁, A₂ and B₁ horizons, and in the case of Shouldice there is actually an accumulation in the A₂ horizon.

The high replaceable calcium content of the B₂ horizon of the various profiles is attributed to calcium carbonate. There is no corresponding increase of replaceable magnesium or sodium-potassium in this horizon in the black and the wooded profiles, but a more or less proportionate increase of magnesium is noticed in the brown soils.

The following conclusions are drawn from the preceding discussion.

(1) In the case of the wooded profiles the leaching processes have been more severe in the *A* horizons than in the black soils and have actually removed a part of the replaceable bases. There has also been a removal of the complex itself as the base-holding capacity is lower in these horizons than it is in the *B*₁ horizons of the wooded profiles or *A* horizons of the black soils.

(2) The bases that have moved in the black and brown soils were in the form of salts. There is no pronounced depletion of replaceable bases in the *A*₂ horizon indicated in these two soil groups. There is also very little difference in the base-holding capacity between the *A*₂ and *B*₁ horizons in these soils, so there has been no removal or disintegration of the complex. Although magnesium and sodium-potassium may have moved down with calcium as salts, their greater solubility prevented them from precipitating in the *B*₂ horizon.

(3) There has been less movement of water-soluble calcium and magnesium in the brown soils, as there is some carbonate found throughout the profile. The somewhat proportionate increase of magnesium in the *B*₂ horizon here indicates that the conditions have been too dry to carry it farther down even though it is more soluble.

All the profiles studied have replaceable calcium, magnesium and sodium-potassium in normal proportions. In a normal soil the replaceable bases are in the following proportions: calcium 75-90%, magnesium 10-25% and a small percentage of sodium-potassium. The Fort Vermilion profile shows a greater content of replaceable magnesium and sodium potassium than is found in the other profiles.

The soils of Alberta also compare very favorably with some of the other soils reported. Bray (2) worked with a number of Illinois soils and found the replaceable bases to be less than 33 M.E. per 100 gm. of soil. Page and Williams (11) determined the replaceable bases in the surface soil of the Broadbalk field at Rothamsted and found them to be between 12 and 16 M.E. In Alberta the replaceable base content of the *A* horizon is between 7.2 and 60 M.E.

In studying the degree of saturation of the various horizons it was found that the calcium chloride method did not give very consistent results. By this method most of the soils showed a base-holding capacity less than the amount of replaceable bases. This is probably due to removal of electrolytes by solution, and of complex by hydrolysis, in the course of leaching and washing. This method is not very satisfactory for demonstrating the small degree of unsaturation found in these soils. However, in samples in which the degree of unsaturation is more pronounced, this method is fairly satisfactory, as, for example, in the cases of the *A*₂ horizon at Fort Vermilion, the *A*₂ horizon at Ksituan and the *B*₁ horizon at Breton.

The most pronounced difference in the degree of saturation is noticed in the *A*₂ horizon. In the black soil this horizon is about 95% saturated in the Edmonton profile, and about 90% in the Hobbema. In the wooded profiles it is between 55% at Fort Vermilion and 88% at Wabamun. This is not a

high degree of unsaturation although the soil has a very leached appearance. This small degree of unsaturation accounts for the more or less neutral reaction of the wooded soils ($\text{pH} > 5$). From this it could be assumed that the aluminosilicate complex has been removed from the A_2 horizon of the wooded soils as a colloidal solution and not by disintegration into oxides of aluminium, silicon and some iron. This is considered to be the case, as the conditions are not very acid, so that very little disintegration of the complex should occur. However, it is probable that the complex enters a more dispersed state by being made more acidic. The accumulation of silicon dioxide in the A_2 horizon is attributed to disintegration of feldspar rocks into base-exchange complex and silica. Truog and Chucka (15) have found this to occur in weathering of feldspars.

The A_0 horizon of the wooded and the A_1 horizon of the black soils show about the same degree of both hydrolytic and replaceable acidity. There is considerably more hydrolytic acidity than replaceable acidity in this horizon, which shows that considerable hydrogen has been taken in. There is, however, no great acidity, the pH of this soil being above 6 as a rule.

The B_1 horizon of the black soils shows very little hydrolytic or replaceable acidity. On the other hand, the A_2 horizon from Ksituan and the B_1 horizon from Breton and Wabamun all show a fair degree of hydrolytic acidity. This presence of hydrolytic acidity in the A_2 and B_1 horizons of the wooded profiles is probably due to precipitation of colloids brought down in the course of leaching. These colloids have some hydrolytic acidity, probably slightly more than those remaining in the upper horizons. However, it is not necessary that all the hydrolytic acidity be neutralized in order to precipitate these colloids. This may account for the presence of this less active acidity in the A_2 and B_1 horizons. These colloids are organic and inorganic, the former being indicated by an increase in nitrogen content.

From the above it could be assumed that in case of the black soils, the A_2 and B_1 horizons have been subjected to less acid conditions than A_1 . Apparently as much hydrolytic acidity has been found in the A_1 horizons of the black soils as in the wooded, but it did not permeate the lower horizons to as great an extent.

The lack of hydrolytic acidity in the brown soils is accounted for by less leaching away of bases, and less production of organic acids.

A very important point is brought up by Gedroiz (3) with respect to leaching. He states that a complex will actually absorb H ions from pure water, with subsequent leaching away of bases, *i.e.*, hydrolysis of the complex will occur. Thus it may be possible that the acidity found in more humid regions is derived, not only from acid leaching, but also from excessive leaching.

It is seen from Table II that only the wooded soils show any indication of replaceable acidity. Thus, according to the Gedroiz system of classification, they fall into the podsol group. It also appears that the black soils are approaching a podsol condition as eventually acidity will pass from a hydrolytic to a replaceable stage.

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STUDIES ON HOMOGENEOUS FIRST ORDER GAS REACTIONS

III. THE DECOMPOSITION OF PARALDEHYDE¹

By C. C. COFFIN²

Abstract

The gaseous decomposition of paraldehyde to acetaldehyde has been studied from points of view already outlined. The reaction, which was followed by increase of pressure at constant volume, is homogeneous, accurately first order, and is presumably uncatalyzed. Its velocity has been measured between 209° and 270°C. at initial pressures of from 1.18 to 52.0 cm. of mercury. It goes to completion under these conditions of pressure and temperature at a rate which is independent of the total pressure and of the partial pressures of paraldehyde, acetaldehyde and mercury vapor. The activation energy is 44160 cal. per mol.

The velocity constants are given by the equation $\ln k = 34.83 - \frac{44160}{RT}$. The bearing of the data on the probably trimolecular reverse reaction as well as on work already reported is discussed.

Introduction

A systematic investigation of homogeneous first order gas reactions is being carried out in this laboratory (1, 2, 3). The main object of the work is to determine to what extent activation energies, as determined from temperature coefficients of reaction rate, are characteristic of molecular structure and how far they may be regarded as quantitative measures of bond stability. It appears also (3) that this work may lead to interesting information regarding the distribution of intramolecular energy. The present paper deals with the gaseous decomposition of paraldehyde to acetaldehyde—a reaction which has been found to go smoothly to completion at a measurable and reproducible velocity over a conveniently attained temperature range. The reaction is homogeneous and monomolecular. It is eminently suited for manometric measurements as the pressure increases 300%. It is of the first order and exhibits no change in velocity over the pressure range investigated, *viz.*, from total pressures of about 1 to 150 cm. of mercury. The reaction is being studied at lower pressures in an attempt to determine the number of squared terms involved in the activation process, and at higher pressures with a view to measuring the velocity of the probably trimolecular reverse reaction. The experiments are also being extended to include polymers of other aldehydes in order to obtain comparisons of the *E*'s and *A*'s of the Arrhenius equation (*cf.* 3). It is to be noted that a paraldehyde decomposition is essentially the change of three ethereal to three carbonyl oxygen molecules as against the one ethereal to one carbonyl oxygen shift of the previously reported ester decompositions.

Although some 120 experiments on the decomposition of paraldehyde have been made to date, only about 55 are concerned with the non-catalytic thermal reaction under conditions where the decomposition is complete, *i.e.*, at pressures

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low enough to allow ordinary manometric methods to be used. Only these latter runs are reported here as the measurement of equilibria and reaction velocities (catalyzed and non-catalyzed) at higher pressures still presents numerous difficulties.

Experimental

Apparatus and Technique

The first 21 runs were carried out in the apparatus already described (3). The remaining 34 were made in the apparatus represented in Fig. 1, which is

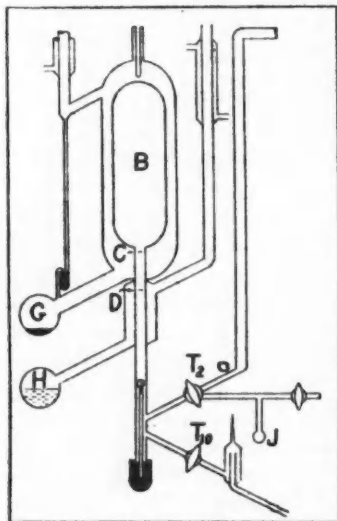


FIG. 1. Diagram of apparatus.

self-explanatory. Xylene was boiled in *H* to prevent condensation of paraldehyde in the manometer when the mercury surface was held at *D*.

A large oil manometer was added beside *M*₃ (2, Fig. 1) for more accurate determination of the lower pressures on the boiling mercury in *G*.

Purification of the Paraldehyde

Two different samples of paraldehyde were used in these experiments. One was obtained by repeated fractional distillation of good commercial paraldehyde. The other was fractionated from c.p. paraldehyde which had been refluxed over, and distilled from, a large excess of metallic sodium. No difference in the decomposition velocities of the two samples was detected and it is therefore believed that no catalytic impurity was present in either.

The Product of the Reaction

As the pressure exactly tripled during each run no great pains were taken to obtain further proof that the product of the reaction was acetaldehyde. After several of the higher pressure runs however the mercury in *N* was lowered to *T*₁₀ and the reaction products were condensed through the three-way tap *T*₂ into the previously evacuated small bulb *J* by means of carbon dioxide and ether. The vapor pressure of the liquid in *J* when measured shortly after condensation was found to be identical with that of acetaldehyde in that it indicated a boiling point under atmospheric pressure of about 20°C. On standing, the vapor pressure invariably fell and the melting point rose, affording further evidence that the original liquid in the bulb was acetaldehyde which slowly polymerized to paraldehyde.

Results

As the reaction is of the type $A \rightarrow 3B$ the partial pressure of paraldehyde at time *t* is given by $\frac{1}{3}(3P_0 - P)$ where *P*₀ is the paraldehyde pressure at the

beginning and P is the total pressure at time t . The monomolecular velocity constant is therefore given by the equation

$$k = \frac{1}{t} 2.303 \log \frac{2P_0}{3P_0 - P}$$

As before (2, 3), the expression $\log \frac{2P_0}{3P_0 - P}$ was plotted on a large scale against time and the velocity constant was determined for each run from the slope of the best straight line drawn through the points. For any one run all these points invariably fall on a perfectly straight line. For different runs, however, carried out under as nearly as possible identical conditions, the slope of these lines varied somewhat as may be seen from the constants of Tables I and II. This suggests that the reaction is sensitive to traces of some catalyst which is present in different amounts in different runs and which, as other experiments indicate, is introduced with the mercury.

In an attempt to identify this supposed catalyst a few runs were made in the presence of air, water vapor and stopcock grease. The velocity constants were usually slightly greater than those obtained in the absence of these substances although the observed acceleration was not in proportion to the amount of impurity added.

In practically all runs the observed final pressure agreed well within 1% with that calculated from the weight of paraldehyde taken, so that it is immaterial which value is used as $3P_0$. This is shown in Table I where both values are listed for a few typical runs at three different temperatures. Column 1 shows one-third of the observed final pressure, column 2 the value of P_0 calculated by the ideal gas laws from the weight of paraldehyde taken and column 3 the velocity constants. In the lower temperature runs where it was possible to obtain P_0 by a short extrapolation it was always found that the observed and calculated values agreed within the limits of error of the extrapolation.

That the specific reaction rate is independent of pressure is well shown in Table II which summarizes the data of the thermal decompositions at seven different temperatures. The initial pressures listed are one-third the observed final pressures and were used in calculating the accompanying constants. At no temperature do the constants show any definite drift as the pressure increases so that within this pressure range the

TABLE I
OBSERVED AND CALCULATED INITIAL PRESSURES*

Final press. 3	P_0 calc.	$k(\text{sec}^{-1})$
$T = 519.3^\circ\text{A.}$		
17.74	17.85	3.06×10^{-4}
24.60	24.88	2.96×10^{-4}
22.87	22.99	2.88×10^{-4}
$T = 526.8^\circ\text{A.}$		
14.12	14.22	5.41×10^{-4}
15.19	15.40	5.15×10^{-4}
8.92	9.00	5.31×10^{-4}
30.73	30.91	5.11×10^{-4}
10.94	10.96	5.52×10^{-4}
$T = 534.9^\circ\text{A.}$		
15.47	15.48	0.904×10^{-3}
22.87	23.04	1.00×10^{-3}
6.79	6.82	1.01×10^{-3}
19.69	19.78	0.920×10^{-3}
5.08	5.07	1.09×10^{-3}

* In cm. of mercury.

TABLE II
 SUMMARY OF THE DATA OF THE THERMAL DECOMPOSITIONS

Temp.	P_0	$k \times 10^6$	Temp.	P_0	$k \times 10^4$	Temp.	P_0	$k \times 10^3$
482.1	12.94	1.11	519.3	1.36	3.06	534.9	3.07	1.09
	20.61	1.05		1.70	2.99		5.02	1.04
	24.96	1.23		1.80	3.34		5.08	1.09
	Mean	1.13		5.90	2.99		5.40	1.05
501.9	5.37	6.31		8.06	3.14		6.79	1.01
	14.56	6.15		10.17	3.06*		7.50	1.07
	16.80	6.29		11.24	3.04*		15.47	0.904
	20.49	6.54		13.57	3.00		15.83	1.09
	20.51	6.59		17.74	3.06		19.69	0.920
	20.97	6.37		20.95	3.00		22.87	1.00
	Mean	6.34		22.87	2.88		37.47	0.973
		$k \times 10^4$		24.60	2.96		Mean	1.02
512.2	10.25	1.66*		Mean	3.05			
	14.84	1.66	526.8	1.18	5.65	542.8	3.23	1.96
	21.78	1.55		1.30	5.49		3.76	1.95
	25.76	1.63		2.42	5.82		6.59	1.91
	36.50	1.73		4.18	5.33		7.10	1.94
	38.94	1.56*		8.92	5.31		18.04	1.88
	52.00	1.50		9.67	5.44*		Mean	1.93
	Mean	1.61		10.94	5.52			
				13.52	5.56			
				14.12	5.41			
				15.19	5.15			
				30.73	5.11			
				Mean	5.44			

reaction is strictly first order. The italicized constants in Table II were obtained in the presence of mercury vapor saturated at the temperature of

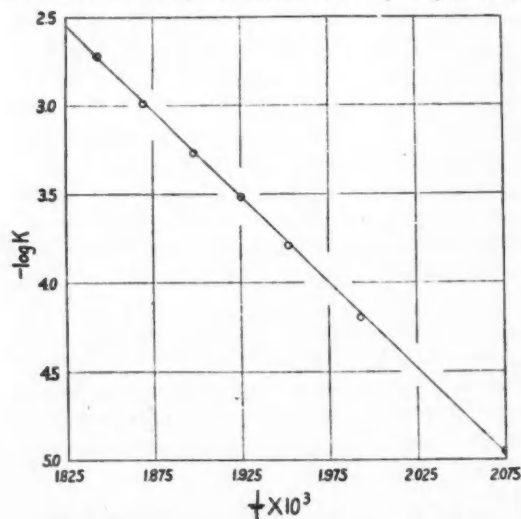


FIG. 2. Graph showing relation between $-\log k$ and $\frac{1}{T}$.

the reaction tube. The others were obtained in the presence of mercury vapor saturated at the temperature of boiling xylene, *i.e.*, at 1.7 mm. pressure. It is evident that mercury vapor between 0.17 and 10.00 ($T = 534.90^\circ$ A.) cm. pressure is without influence on the rate of decomposition.

Homogeneity of the Reaction

In Table II the values marked with an asterisk are the results of experiments in which the glass surface in contact with the paraldehyde vapor was enor-

mously increased by packing the reaction chamber with glass wool. It is evident that the extent of glass surface is without influence on the rate of the reaction which must therefore take place homogeneously throughout the gas.

While the final pressures of the decompositions in the unpacked bulb were found to remain constant to within 0.1 cm. for an indefinite time even at the highest temperatures, the final pressures of the runs in the packed bulb invariably decreased at a rate which was apparently dependent on the temperature, the total pressure and the amount of glass wool in the reaction chamber. A decrease of as much as 10 cm. in 24 hr. was observed at 519°A., with an initial aldehyde pressure of about 30 cm. After such an experiment the inside of the cold tubing leading to the reaction chamber was covered with yellow oily droplets of a high boiling liquid having an odor reminiscent of geraniol. The alkaline surface of the glass wool evidently brings about some sort of aldehyde condensation.

The Energy of Activation

In Fig. 2, $\log k$ is plotted against $1/T$. The temperatures and average constants of Table II were used. The slope of the straight line corresponds to an activation energy of 44,160 cal. per mol. Velocity constants are given by the equation $\ln k = 34.83 - \frac{44160}{RT}$.

Discussion

It is of interest to consider the reverse reaction in the light of the data already available. Roozeboom (9) and Hollmann (6) found the "natural" critical temperature of the system to be 218–221°C. and the equilibrium concentration of paraldehyde to be 11 mol. per cent. The reverse reaction thus takes place at a measurable rate within the temperature range over which the paraldehyde decomposition has been investigated, if the pressure is sufficiently high. Moreover, as the decomposition of paraldehyde is homogeneous the reverse reaction must likewise be homogeneous since a catalyst (e.g., a glass wall) cannot change an equilibrium. Unfortunately the critical pressures and absolute concentrations were not measured so that the specific velocity of the presumably third-order formation of paraldehyde cannot be estimated from that of its first order decomposition. Even if this were possible such an extrapolation of low pressure data would be anything but reliable as the velocity (8) and order (10) of a reaction may change considerably with pressure.

The activation energy (E_1) of a reaction and that (E_2) of its reverse are related to the heat of reaction Q by the equation $Q = E_1 - E_2$.

Cooper (4, 5) in this laboratory found the heat of the reaction, 3 acetaldehyde \rightarrow paraldehyde, to be 19,400 ($\pm 1\%$) cal. per mol. A less reliable value of something over 20,000 cal. is obtained from the heats of combustion (7). Taking Q as 19,400 and E_1 as 44,200, E_2 is found to be 24,800 cal. per mol of paraldehyde formed—an activation energy of the order of magnitude to be expected for a trimolecular reaction occurring under the above conditions of pressure and temperature.

As stated above further experiments along these lines are in progress.

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THE ELECTRIC MOMENT OF HYDROGEN PEROXIDE¹

By E. P. LINTON² AND O. MAASS³

Abstract

The electric moment of hydrogen peroxide was found to be 2.13×10^{-18} in dioxan. Comparative measurements made on the electric moment of water gave 1.90×10^{-18} in dioxan. Measurements made with both water and hydrogen peroxide in ether gave lower values for their electric moments, and the reason for this is discussed. The structure of hydrogen peroxide is discussed, and the electric moment shown to be in agreement with all data as determined in this laboratory in indicating the presence of a co-ordinate co-valent bond.

The dielectric constant of a substance has always been considered an important physical property. In the last few years considerable advances have been made in the field of molecular structure due to the work of Debye (4, 5) and Smyth (11, 12), these investigators having made possible the calculation of the electric moment of a molecule from dielectric-constant data.

Theoretical

There are two general methods for finding the electric moment of the molecule. The first, due to Debye, consists of measuring the temperature coefficient of the dielectric constant of a gas, while the second consists of the measurement of the dielectric constant of dilute solutions of a substance in a non-polar solvent. The second method was used in this investigation as hydrogen peroxide is not stable in the gaseous state.

As the theory of dielectric polarization has been discussed elsewhere (4, 5, 11, 12) only the equations immediately necessary will be given here. The molar polarization of a substance in which the molecules are free to assume a random orientation, as in the gaseous state, is given by

$$P = \frac{\epsilon - 1}{\epsilon + 2} \frac{M}{d} = \frac{4\pi}{3} \gamma N + \frac{4\pi N}{9\kappa} \frac{\mu^2}{T}, \quad (1)$$

where ϵ = dielectric constant, M = molecular weight, d = density, N = number of molecules in a gram molecule, γ = molecular polarizability, κ = molecular gas constant, T = absolute temperature, and μ = electric moment of a molecule.

The term $\frac{4\pi}{3} N\gamma$ is the polarization due to shifts induced in the molecule by the external field, and is usually calculated as the molar refraction of light at infinite wave-length. The expression $\frac{4\pi}{3} N\gamma$ includes both the electronic shifts induced and the shifts induced in the atom or radicals. Therefore $\frac{4\pi}{3} N\gamma = P_E + P_A$.

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The expression $\frac{4\pi}{9\kappa} \frac{\mu^2}{T}$ gives the moments of the molecules oriented by the electric field equal to P_M . The total polarization is then, $P = P_E + P_A + P_M$.

The above equations apply to pure substances in the gaseous state. In liquids, however, the molecules are so close together that if they contain doublets they affect one another so that their freedom of motion is influenced. If the molecules of a polar liquid are separated from one another by molecules of a non-polar liquid, the polar molecules should behave much as in the gaseous condition. The molar polarization of the mixture of the two liquids is given by

$$P_{12} = \frac{\epsilon - 1}{\epsilon + 2} \frac{C_1 M_1 + C_2 M_2}{d} = C_1 P_1 + C_2 P_2, \quad (2)$$

where C_1 and C_2 , M_1 and M_2 , P_1 and P_2 are the molar fractions, molecular weights and polarizations of the two components, and

$$C_1 = 1 - C_2, \quad P_2 = \frac{P_{12} - P_1}{C_2} + P_1. \quad (3)$$

Thus the polarizations of the polar liquid are calculated at various concentrations. These polarizations are plotted against mole fraction and the value of P_2 at infinite dilution (*i.e.*, P_{00}) is obtained.

$$P_{00} = P_E + P_A + P_M \quad P_M = P_{00} - MR_{00}, \\ = 0.0127 \times 10^{-18} \sqrt{P_{00} - MR_{00} T},$$

neglecting P_A which is very small for water or hydrogen peroxide.

In the experiments carried out by the authors, ether and dioxan were used as the non-polar solvents. Ether has a higher dielectric constant than the ordinary non-polar solvents and it is possible that hydrogen peroxide would behave quite differently in ether than in an ordinary non-polar solvent such as dioxan. Dioxan has been used as a solvent by Williams (15) and Smyth (14). They considered that it compared favorably with benzene or hexane as a solvent for the determination of electric moments. Dioxan is miscible with water and hydrogen peroxide in all proportions and dissolves many substances which are insoluble in the ordinary non-polar liquids. The electric moment of water has been determined by several investigators (7, 9, 15, 16) both in the liquid and gaseous state. In order to make certain that the method gave correct results the experiments were carried out on dilute solutions of water in ether and dioxan. In this way it was possible to compare the results for both water and hydrogen peroxide in the two solvents.

Apparatus

The resonance method for the measurement of the dielectric constant has been described in detail (8). A power tube was used in the variable oscillator to overcome any conductivity effects in the solutions. The dielectric cell was made of pure block tin and was described previously (8).

In order to avoid as much as possible the loss of ether by volatilization the dielectric-constant measurements were carried out at 10°C. in the case of the ether solutions. The solutions were made up by adding a weighed quantity

of hydrogen peroxide or water to a known quantity of ether or dioxan. Additional amounts of hydrogen peroxide and water were added in order to give solutions of increasing concentration. Smyth's value (14) of 2.306 was taken as the dielectric constant of pure dioxan at 25°C. The dielectric constant of pure ether at 10°C. was taken to be 4.52.

Purification of Materials

The dioxan was obtained from the Carbide and Carbon Chemicals Corporation. It was recrystallized twice and distilled from sodium: boiling point, 101.3°C. at 760 mm. pressure; density, 1.031 at 25°C.; refractive index, 1.4203; dielectric constant, 2.306.

The ether was shaken with its own volume of water five times, dried over calcium chloride and distilled from sodium: density, 0.726; dielectric constant, 4.52 at 10°C.

Hydrogen peroxide was prepared in the usual way by distillation and concentration of the 30% crude material. The hydrogen peroxide (96%) was recrystallized twice to give peroxide approximately 99% pure. This sample was used in making up the solution by adding it from a weight pipette.

The ordinary distilled water of the laboratory was used to make up the dilute solutions of water.

Results

Table I shows the experimentally determined dielectric constants and the density of the liquid mixtures, together with the values of the polarization P_{12}

TABLE I
DIELECTRIC CONSTANTS, DENSITIES AND POLARIZATIONS

Mole fraction water	Dielectric constant	Density	Polarization		Mole fraction water	Dielectric constant	Density	Polarization	
			P_{12}	P_2				P_{12}	P_2
Water in dioxan, 25° C.					Hydrogen peroxide in ether, 10° C.				
0.000	2.306	1.0311	25.85	25.85	0.0000	4.52	0.726	55.0	55.0
0.043	2.542	1.0310	28.05	78.00	0.0312	4.97	.735	56.35	98.3
0.078	2.790	1.0300	29.85	77.50	0.0512	5.32	.742	57.25	99.0
0.114	3.120	1.0290	32.15	79.60					
Water in ether, 10° C.					Hydrogen peroxide in ether, 0° C.				
0.0000	4.52	0.726	54.95	P_1	0.1022	6.6	0.769	59.3	99.8
0.0282	4.74	.727	55.35	69.8	0.1950	8.7	.791	60.2	82.5
0.0430	5.07	.729	55.80	73.6	0.3230	13.2	.834	58.8	67.2
					0.4360	18.0	.892	53.8	52.8
					0.5740	26.9	.954	48.2	43.3
					0.6540	33.8	.997	44.0	38.3
					0.7500	44.3	1.071	38.2	32.7
					1.0000	91.0	1.460	22.6	22.6
Hydrogen peroxide in dioxan, 25° C.									
0.00000	2.306	1.031	25.85	25.85					
0.02104	2.451	1.034	27.30	94.2					
0.04105	2.570	1.036	28.50	89.4					
0.06100	2.722	1.039	29.70	88.2					

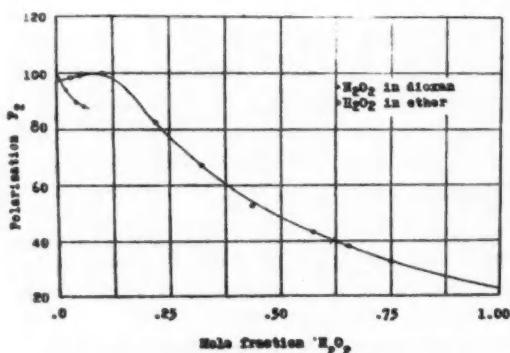


FIG. 1. Relation between polarization of hydrogen peroxide and mole fraction in solvent.

TABLE II
MOLAR REFRACTION, POLARIZATION AT INFINITE DILUTION AND MOMENT
OF WATER AND HYDROGEN PEROXIDE

	MR_D	P_∞	$P_\infty - MR_D$	$\mu \times 10^{18}$
Water in dioxan	3.7	79.1	75.4	1.90
Water in ether	3.7	67.2	63.5	1.71
Hydrogen peroxide in dioxan	5.6	100.2	94.6	2.13
Hydrogen peroxide in ether	5.6	97.7	92.1	2.06

Discussion

The values given for water and hydrogen peroxide neglect the atomic polarization P_2 . This term may be disregarded when the dielectric constant of the liquid is large and its molecule contains only a small number of atoms. The value for the moment of water in dioxan agrees very closely with the values of other investigations (7, 9, 15, 16).

The value found for water in ether is considerably lower than the value found in other solvents. The measurements in ether solution were not as accurate as the measurements in the other experiments due to the small solubility of water in ether, making the extrapolation of P_1 to infinite dilution uncertain. The difference in the values of the electric moment of water may be due to the fact that the measurements in ether were carried out at a lower temperature in order to avoid the loss of ether by volatilization. The value found for the temperature coefficient of the dielectric constant of water shows that the dielectric constant increases rapidly at temperatures below 15°C. (6). This large increase is probably due to a change in the association of water at these temperatures. Thus, at 10°C., the temperature at which these determinations of the electric moment of water in ether were carried out, the value found for the electric moment of water may be considerably lower than the value found at 25°C.

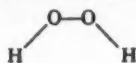
calculated by means of Equation (2). P_1 is calculated from P_∞ as shown in Equation (3).

The values for P_∞ are obtained by extrapolation of the values of P_1 to infinite dilution. These extrapolations for hydrogen peroxide are shown in Fig. 1.

Table II shows the molar refraction, polarization at infinite dilution and moment of water and hydrogen peroxide.

On the other hand, the higher dielectric constant of ether should cause the water to be less associated than it is in dioxan solution (13). Evidently the temperature effect is considerably larger than the effect due to the large dielectric constant of the ether. As was to be expected, the electric moment of hydrogen peroxide is greater than the moment of water. As in the case of water, the moment in the ether solution is less than in the dioxan solution, probably for the same reasons.

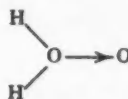
The high electric moment of water has been explained by assuming an unsymmetrical arrangement for the atoms in the molecule. Debye (8) has given a detailed mathematical analysis of the subject, and concludes that the molecule which best fits the facts is that in which the valence bonds are at an angle of 64° with one another. Hydrogen peroxide may have any of the following formulas:—



I



II



III

All physico-chemical measurements, such as parachor (1), molecular refractive power (3), and high dielectric constant (2), indicate that III is the correct form. The ease with which hydrogen peroxide loses an oxygen atom and forms water is in agreement with this.

The high electric moment of hydrogen peroxide is against Formula I, as this formula would have a very small moment in comparison with the others.

Formula III contains a co-ordinate co-valent bond which usually gives molecules large electric moments (10). Therefore the high electric moment of hydrogen peroxide points to Formula III as being the correct one. To sum up, all data as determined in this laboratory are in favor of this structure.

The calculation of an angle between valence bonds is left for the present.

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PROPAGATION OF ULTRASOUND IN SOLID CYLINDERS (TRANSVERSE WAVES)¹

BY R. RUEDY²

Abstract

The expression giving the phase velocity c with which flexural waves pass through long solid rods is deduced for frequencies varying between zero and over 1,000,000 cycles per sec. and rods of any diameter. As the frequency increases, the velocity c increases gradually from very low values toward $c^2 = mE/2s(m+1)$, reached when the wave-length is much smaller than the diameter of the rod. Published experimental results for transverse waves are in good agreement with the theory given. In general at least four effects enter into the propagation of ultrasound through solid cylinders: first, longitudinal waves, for which the phase velocity decreases toward c as the frequency increases; second, transverse waves, for which the phase velocity increases toward c as the frequency increases; third, pure radial waves at certain frequencies; fourth, resonance effects between the different types of waves, which, on account of the mechanical coupling existing between them, change the natural period of vibration of the rod without affecting the velocity.

Introduction

When attempts are made to send purely longitudinal waves of high frequency through a solid rod, other types of motions besides those originally excited invariably appear (1-3, 6-8). That the solid particles set into vibration along the axis show at the same time motions perpendicular to this direction, has of course its reason in the contractions and expansions which a solid rod undergoes when transmitting longitudinal waves, the changes in thickness being given by Poisson's ratio. There is at no moment in the rod a cross-section in which the particles move only parallel to the axis, and the amplitude of the radial displacement increases from the interior toward the surface. On the other hand, the longitudinal and radial oscillations of the particles may be considered as components in part of a more general type of wave, the transverse or flexural kind, and experience has shown that over a certain range of frequencies flexural waves are very strongly excited, especially in cylindrical rods the diameter of which does not exceed a few millimetres. Both waves may be present at the same time and, in rods of finite length, produce their own set of stationary dust figures. It may become impossible to say to which wave a given figure belongs unless it is known in what way the velocity c of the wave changes with frequency.

The very high frequencies at which confusion is most likely to occur are of great practical importance as they belong to the range of broadcasting waves. Good stations maintain their carrier frequencies within very narrow limits, in some cases less than ten cycles, by means of oscillating rods or disks. It is, therefore, desirable to know the velocity of propagation of longitudinal and flexural waves from zero frequency to well over 1,000,000 cycles per sec., to ascertain whether the presence of both longitudinal and transverse waves leads to the formation of wave-groups having a higher velocity of propagation

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than the waves themselves, and to study the possible effects which the second wave-system might have on the natural frequencies of vibration of a rod of standard size in which longitudinal waves are excited for calibration purposes. Means for suppressing a certain type of wave are also of interest.

The Velocity of Transverse or Flexural Waves

When a flexural wave is sent through a cylindrical rod, the general equations of the motion of the particles are, in cylindrical co-ordinates r, θ, z ,

$$u = U(r) \cos \Theta e^{i(\gamma z + pt)} \quad v = V(r) \sin \Theta e^{i(\gamma z + pt)} \quad w = W(r) \cos \Theta e^{i(\gamma z + pt)}$$

u being the displacement in the direction of the radius, v that along θ , or the tangential displacement, and w the displacement parallel to the long axis z . The frequency f appears in $p = 2\pi f$ and the wave-length in $\gamma = 2\pi f/c$. The equations of motion furnished by the theory of elasticity lead to the following solution (4, §. 214),

$$U(r) = A \frac{\partial J_1(hr)}{\partial r} + B\gamma \frac{\partial J_1(hr)}{\partial r} + C \frac{J_1(hr)}{r}$$

$$V(r) = -A \frac{J_1(hr)}{r} - B\gamma \frac{J_1(hr)}{r} - C \frac{\partial J_1(hr)}{\partial r}$$

$$W(r) = A i \gamma J_1(hr) - B i k^2 J_1(hr)$$

with

$$k^2 = \frac{p^2}{c_0^2} \frac{(m+1)(m-2)}{m(m-1)} - \frac{p^2}{c^2} = \frac{p^2}{c^2} \left(\frac{c^2}{c_0^2} \frac{(m+1)(m-2)}{m(m-1)} - 1 \right)$$

$$k^2 = \frac{p^2}{c_0^2} \frac{2(m+1)}{m} - \frac{p^2}{c^2} = \frac{p^2}{c^2} \left(\frac{c^2}{c_0^2} \frac{2(m+1)}{m} - 1 \right).$$

$E/s = c_0^2$, the square of the velocity of longitudinal waves of low frequency, and $2 < m < 5$, where m designates the reciprocal of Poisson's ratio. J_1 is the Bessel function of the first kind of order one, and as it becomes equal to zero when the argument is zero, points lying on the central axis evidently do not move along the axis, but only in a plane perpendicular to it, in contrast with the conditions found for a longitudinal wave. Moreover because the traction must vanish at the surface the following equations must be satisfied over the entire length of the cylinder, wherever $r = a$, the radius of the cylinder:

$$\frac{u}{a} + (m-1) \frac{\partial u}{\partial a} + \frac{1}{a} \frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z} = 0$$

$$\frac{v}{a} - \frac{1}{a} \frac{\partial u}{\partial \theta} - \frac{\partial v}{\partial a} = 0$$

$$\frac{\partial u}{\partial z} + \frac{\partial w}{\partial a} = 0$$

or

$$i\gamma U(a) + \frac{\partial W}{\partial a} = 0$$

$$\frac{U}{a} + \frac{V}{a} - \frac{\partial V}{\partial a} = 0$$

$$\frac{U}{a} + \frac{V}{a} + (m-1) \frac{\partial U}{\partial a} + i\gamma W = 0$$

These boundary conditions yield therefore three equations from which A, B and C may be eliminated, and the phase velocity $c = p/\gamma$ calculated in terms of

the radius a and the frequency f . After the derivation has been carried out the three boundary conditions become

$$\begin{aligned} 2hJ_2(ha)A + 2\gamma kJ_2(ka)B - (2kJ_2(ka) - k^2aJ_1(ka))C &= 0 \\ 2\gamma aJ_1(ha)A + (\gamma^2 - k^2)aJ_1(ha)B + \gamma J_1(ka)C &= 0 \\ A[(m-2)(hJ_2(ha) - h^2aJ_1(ha)) - (\gamma^2 + h^2)aJ_1(ha) \\ + B\gamma(m-2)(kJ_2(ka) - k^2aJ_1(ka)) \\ - (m-2)kJ_2(ka)C &= 0, \end{aligned}$$

and the final result is obtained in the following form:

$$\begin{aligned} 2ka \frac{J_2(ka)}{J_1(ka)} \left[(\gamma^2 - k^2) \left(k^2 + \frac{\gamma^2 + h^2}{m-2} \right) \right] \\ + \frac{J_2(ka)}{J_1(ka)} \left[6\gamma^2 k^2 - \left(k^2 + \frac{\gamma^2 + h^2}{m-2} \right) (2\gamma^2 + (\gamma^2 - k^2)(2 + k^2a^2)) \right] \\ + hk \frac{J_2(ha)}{J_1(ha)} \left[(\gamma^2 + k^2 + 2\gamma^2 k^2 a^2) - ka \frac{J_2(ka)}{J_1(ka)} (6\gamma^2 - (\gamma^2 - k^2)) \right] \\ - ka \left[2\gamma^2 k^2 - \left(k^2 + \frac{\gamma^2 + h^2}{m-2} \right) (\gamma^2 - k^2) \right]. \end{aligned}$$

When for a given frequency f , a certain phase velocity c is assumed, the values γ , k , $J_2(ka)/J_1(ka)$, h , $J_2(ha)/J_1(ha)$ follow immediately and on introducing them into the equation, the correctness of the choice of c can be tested from case to case. Three trials suffice in general for extrapolating to the correct value. For preparing charts showing, for a certain material, how the velocity of propagation of the transverse waves varies with frequency and diameter of the rod, the simplest procedure is to assume a series of values of c for the same frequency and to make these values fit the equation by the proper choice of a .

Discussion of the Velocity Formula

On account of the nature of the Bessel functions J_2 and J_1 a discussion of the general solution would be difficult and not necessarily useful; moreover quite different velocities of propagation would likely be found for the same frequency. For the physical problem at hand, account must be taken of the fact that for thin rods and low frequencies both experiment and theory show that c is given by the equation

$$c^2 = \pi a c_0 f,$$

that is, it varies with frequency according to a square law, being very much smaller than c_0 , the velocity of longitudinal waves at low frequencies. Only those solutions of the general equation will therefore be retained which go over into the values known to be valid for ordinary frequencies, and discontinuous changes of the velocity will be excluded unless they are to be expected in a certain range on physical grounds. This means that starting with frequencies in the neighborhood of zero, both h and k are imaginary quantities. (The letters h and k will, however, be retained to designate the real part as if what had been called h and k up to this point had actually been $h' = ih$ and $k' = ik$.) Replacing at the same time part of the symbols h and k by their values, the general solution thus becomes

$$\begin{aligned}
& ka \left(\frac{c^2}{c_0^2} \frac{m+1}{m} \right)^2 + 2ka \frac{J_2(ika)}{J_1(ika)} \left(\frac{c^2}{c_0^2} \frac{m+1}{m} - 1 \right)^2 \\
& + \frac{J_2(ika)}{iJ_1(ika)} \left[2 - 5 \frac{m+1}{m} \frac{c^2}{c_0^2} - \left(\frac{c^2}{c_0^2} \frac{m+1}{m} - 1 \right)^2 (2 - k^2 a^2) \right] \\
& + \frac{hk}{\gamma^2} \frac{J_2(ika)}{iJ_1(ika)} \left[\frac{m+1}{m} \frac{c^2}{c_0^2} - k^2 a^2 + \left(2 + \frac{c^2}{c_0^2} \frac{m+1}{m} \right) \frac{kaJ_2(ika)}{iJ_1(ika)} \right] = 0,
\end{aligned}$$

where h^2 and k^2 are positive quantities both smaller than γ^2 and $k < h$. As the ratio $\frac{J_2(ika)}{iJ_1(ika)}$ has only positive values (Table I) all the terms appearing in the equation become real.

TABLE I
VALUES OF $\frac{J_2(ig)}{iJ_1(ig)}$

g	$\frac{J_2(ig)}{iJ_1(ig)}$	g	$\frac{J_2(ig)}{iJ_1(ig)}$	g	$\frac{J_2(ig)}{iJ_1(ig)}$	g	$\frac{J_2(ig)}{iJ_1(ig)}$	g	$\frac{J_2(ig)}{iJ_1(ig)}$
0.0	0.00	1.0	0.240	2.0	0.433	3.0	0.568	4.0	0.658
0.2	0.05	1.2	0.283	2.2	0.464	3.2	0.589	5.0	0.719
0.4	0.099	1.4	0.324	2.4	0.494	3.4	0.608	10.0	0.854
0.6	0.148	1.6	0.363	2.6	0.520	3.6	0.626	15.0	0.902
0.8	0.195	1.8	0.399	2.8	0.545	3.8	0.642		

An insight into the way in which the velocity c varies with frequency may then be readily obtained by choosing simple fractions of $m/(m+1)$ as values for c^2/c_0^2 , such as $m/18(m+1)$, $m/8(m+1)$ up to $m/2(m+1)$, the largest value for which k remains positive and real. The following expressions correspond then to one another:

$$\frac{c^2}{c_0^2} \quad \frac{c^2}{c_0^2} \frac{m+1}{m} \quad h^2 \quad k^2 \quad \frac{hk}{\gamma^2}$$

$$\frac{m}{x(m-1)} \quad \frac{1}{x} \quad \gamma^2 \left(1 - \frac{m-2}{x(m-1)} \right) \quad \gamma^2 \left(1 - \frac{2}{x} \right) \quad \frac{1}{x} \sqrt{(x-2) \left(x-1 + \frac{1}{m-1} \right)}.$$

When c/c_0 increases both h and k fall, k more quickly than h , from the value unity for very low velocities toward zero. Writing y for $\gamma a = 2\pi f a/c$, the general equation now reduces to:

$$\begin{aligned}
& \frac{\sqrt{1-\frac{2}{x}}}{x^2} y + 2 \left(1 - \frac{1}{x} \right)^2 \sqrt{1-\frac{2}{x}} y \frac{J_2^2}{J_1^2} \left(i \sqrt{1-\frac{2}{x}} y \right) \\
& + \frac{J_2}{iJ_1} \left(i \sqrt{1-\frac{2}{x}} y \right) \left[y^2 \left(1 - \frac{2}{x} \right) \left(1 - \frac{1}{x} \right)^2 - \frac{1}{x} - \frac{2}{x^2} \right] \\
& + \frac{hk}{\gamma^2} \frac{J_2}{iJ_1} \left(i \sqrt{1-\frac{m-2}{x(m-1)}} y \right) \left[\left(2 + \frac{1}{x} \right) \sqrt{1-\frac{2}{x}} y \frac{J_2}{iJ_1} \left(i \sqrt{1-\frac{2}{x}} y \right) + \frac{1}{x} - \left(1 - \frac{2}{x} \right) y^2 \right] \\
& = 0,
\end{aligned}$$

where $\frac{J_2}{iJ_1}(iy)$ means $\frac{J_2(iy)}{iJ_1(iy)}$.

For a very low velocity or a very large x the equation becomes

$$\frac{J_2(iy)}{iJ_1(iy)} = \frac{y}{2},$$

with the solution $y=0=2\pi fa/c$, that is with the lowering of the velocities the frequencies tend even more rapidly toward zero than the velocity. Lessening the thickness of the rod has the same effect. On going to larger values of c and y , the coefficient h remains near unity much longer than k , and even for $x=4$ it has dropped to only 0.94 for $m=3$, and to 0.91 for $m=4$. For different values of m the equations differ from one another only by their last term, which is for $m=3$ (i.e., for tin, or drawn aluminium of 92% purity, or drawn copper with about 0.2% As, as against $m=2.9$ for the pure metals)

$$\frac{\sqrt{(x-2)(x-0.5)}}{x} \frac{J_2}{iJ_1} \left(iy \sqrt{1 - \frac{0.5}{x}} \right) \left[\dots \dots \right],$$

and for $m=4$ (certain types of special glasses)

$$\frac{\sqrt{(x-2)(x-0.67)}}{x} \frac{J_2}{iJ_1} \left(iy \sqrt{1 - \frac{0.67}{x}} \right) \left[\dots \dots \right].$$

In Table II are shown the solutions y for various values of x , and $m=3$, and in Fig. 1 the computed points have been indicated on the experimental curves for aluminium for which approximately $m=3$.

TABLE II
SOLUTION OF THE PHASE VELOCITY EQUATION FOR $m=3$

x	18	8	4	3
c/c_0	0.204	0.307	0.433	0.500
$y=2\pi af/c$	0.5	0.75	1.35	1.86

For $x=2$ the equation becomes equal to zero identically, i.e., for any frequency the velocity is $(m/2(m+1))^{1/2}$, corresponding to the case in which the wave-length of the purely transverse waves is much smaller than the diameter of the rod. Furthermore for $x=1$, we have

$$y - 3 \frac{J_2}{J_1}(y) + \frac{1}{\sqrt{m-1}} \left(1 + y^2 - 3y \frac{J_2}{J_1}(y) \right) \frac{J_2}{iJ_1} \left(iy \sqrt{1 - \frac{m-2}{m-1}} \right),$$

or, for $m=3$

$$y - 3 \frac{J_2}{J_1}(y) + 0.707 \left(1 + y^2 - 3y \frac{J_2}{J_1}(y) \right) \frac{J_2}{iJ_1}(0.707iy) = 0.$$

The question here is whether when k , which has become zero for $c^2 = c_0^2 m/2(m+1)$, passes over to negative values the velocity resulting from the equation continues to change gradually, in other words whether the equation for $x=1$ has a solution for rather small values of y . When y varies from 0 to 3.8317, the ratio $J_2(y)/J_1(y)$ varies from zero to infinity and beyond this point jumps suddenly to large negative values. The equation admits of a solution in the neighborhood of y equals two, that means for finite frequencies. Apparently even if the values of y are restricted to those smaller than 3.83 two different velocities result for each frequency. There is, however, a sudden change in

the type of motion at the point where k has dropped to zero so that it is uncertain whether the higher speeds have any physical significance.

As in the case of the elementary formula $c^2 = \pi a c_0 f$, valid for very thin rods and low frequencies, the velocity at higher frequencies and in thicker rods depends on the product af only, so that theoretically when the velocity c has been found for a certain frequency f and radius a , the same solution ought to apply to a higher frequency in a correspondingly smaller rod.

Motion of Particles

The equation governing the velocity of the transverse waves in a solid cylinder involves the assumption that the motion u along the radius of a particle lying at the distance r from the central axis; v , its motion perpendicular to u , but in the same cross-section; and w , the motion of the point parallel to the axis, are given by the following solution of the general theory:

$$u = \left[A \left(ikJ_0(ikr) - \frac{J_1(ikr)}{r} \right) + B\gamma \left(ikJ_0(ikr) - \frac{J_1(ikr)}{r} \right) + C \frac{J_1(ikr)}{r} \right] \cos \Theta e^{i(\gamma z + pt)}$$

$$v = \left[A \frac{J_1(ikr)}{r} - B\gamma \frac{J_1(ikr)}{r} - C \left(ikJ_0(ikr) - \frac{J_1(ikr)}{r} \right) \right] \sin \Theta e^{i(\gamma z + pt)}$$

$$w = \left[A\gamma iJ_1(ikr) + Bk^2 iJ_1(ikr) \right] \cos \Theta e^{i(\gamma z + pt)}$$

As $iJ_0(ikr)$ and $J_1(ikr)$ are positive and imaginary, and the constants A , B , C real quantities, the exponential $i(\gamma z + pt)$ term furnishes the function sine ($\gamma z + pt$) for real values of u and w , and $\cos (\gamma z + pt)$ for v :

$$u = \left[A \left(\frac{J_1(ikr)}{ir} - kJ_0(ikr) \right) + B\gamma \left(\frac{J_1(ikr)}{ir} - kJ_0(ikr) \right) - C \frac{J_1(ikr)}{ir} \right] \cos \Theta \sin (\gamma z + pt)$$

$$v = \left[-A \frac{J_1(ikr)}{ir} + B\gamma \frac{J_1(ikr)}{ir} + C \left(kJ_0(ikr) - \frac{J_1(ikr)}{ir} \right) \right] \sin \Theta \sin (\gamma z + pt)$$

$$w = \left[A\gamma iJ_1(ikr) - Bk^2 iJ_1(ikr) \right] \cos \Theta \cos (\gamma z + pt).$$

As an example, the elongations in the direction parallel to the length of the

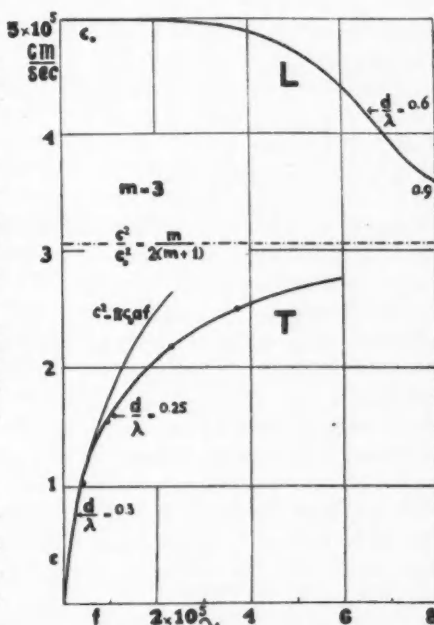


FIG. 1. Velocity of longitudinal (curve L) and transverse waves (curve T with computed points marked) in an aluminium rod 0.4 cm. thick and 35.1 cm. long (6).

rod of the particles lying upon the surface have been computed for an aluminium rod 4 mm. thick (Table III).

TABLE III
VIBRATION w AT THE SURFACE OF AN ALUMINIUM ROD 4 MM. THICK

f per sec.	w
5×10^4	$(-0.69A + 0.199B) \cos \Theta \sin (\gamma s + pt)$
10^5	$(-1.56A + 0.541B) \sin \Theta \sin (\gamma s + pt)$
3.75×10^5	$(-11.83A + 5.62B) \cos \Theta \cos (\gamma s + pt)$

When the experimental conditions are such that for the lowest frequency the greatest elongation is equal to one-half ($A=1$, $B=1$), this value will rapidly increase as higher frequencies are excited in the rod. It is also possible to imagine a case in which the transverse component w vanishes as the number of cycles increases, for instance when $A=1$, $B=2$. All this agrees with the observed fact that flexural waves are relatively strong over a certain range of the higher frequencies.

It is also of interest to compare the way in which the amplitude varies over the cross section in the case of longitudinal as against flexural waves. A particle set into motion by a longitudinal wave vibrates in the direction s according to the equation:

$$w = [-A\gamma J_0(ikr) - Ck J_0(kr)] \sin(\gamma s + pt).$$

Taking again as an illustration an aluminium bar of 6 mm. thickness, it is found that the amplitude of the longitudinal waves scarcely varies along the radius provided that the frequency does not exceed about 150,000 cycles. At 100 kc. for instance the ratio between edge and centre is about 0.98. This means that when one end of the rod is brought into close contact with an oscillating piston, such as is represented by an oscillating quartz plate, the end can adapt itself to the boundary conditions thus imposed without any strains building up over the cross section. At higher frequencies, however, there is a marked drop in the amplitude of w along the radius in the natural state of vibration, and the vibrations forced upon the end cause internal forces of deformation over the cross section, stresses which are relieved by the setting up of flexural and possibly still other forms of vibrations, such as surface waves for instance. The uneven distribution of w will tend to occur at lower frequencies for thicker rods as shown by theory and experiment (2, 6). One method of lessening the strength of transverse waves consists therefore in using as a source a vibrating disk in which the amplitude decreases from the centre toward the edge. Unfortunately a separate disk would have to be used for different ranges of wave-lengths. A thinner rod placed between the source and brought into contact with a thicker rod is likely to serve the same purpose over a limited band of frequencies.

It is by no means certain that all the points of the quartz disks as usually cut and employed for exciting vibrations in a rod move in phase, and when as is sometimes the case (5), a small layer of oil lies between disk and rod, the uneven

distribution of the forces in the sound field facing an oscillating piston surface is likely to complicate matters. However that may be, both transverse and longitudinal waves are as a rule directly set up in the rod and not the flexural waves by way of the forced longitudinal waves. To a certain extent this also applies to the radial waves, so that when a high frequency sound wave acts upon a rod it influences a system which would possess in the uncoupled state three distinct series of natural frequencies of vibration. On account of the unavoidable mechanical coupling existing between the different types of vibration, each natural period of vibration is influenced by the presence of one at least of the other types, so that two periods of resonance exist in each system, one higher and the other lower than the period of the uninfluenced systems. In the place of complete resonance, beats may appear between the two types of vibration (2). The actual effects to be expected depend, however, on the degree of damping which assumes different values in each case, particularly at high frequencies, where moreover the value of E is no longer a constant.

When solutions of the equation of motion are admitted in which the square of the velocity exceeds the value $c_0^2 m/2(m+1)$ so that ik becomes real, the type of motion changes completely; instead of a single cosine wave for w , for instance, there will be two waves:

$$w = A \gamma i J_1(ihr) \cos \Theta \cos (\gamma s + pt) - B k^2 J_1(kr) \cos \Theta \sin (\gamma s + pt).$$

There is as yet no experimental evidence of such a sudden change.

Possible Group Velocities

As it seems to be very difficult to prevent the appearance of flexural waves when high frequency sound waves act upon solid bodies, and as both types cause similar longitudinal and radial motions w and u at the surface of the rod, it is possible that the two systems form trains of waves, and that the velocity of the wave-groups is higher than the phase velocities. It is known that when two slightly different frequencies are exciting transverse waves in a bar, they will form a train of waves of the equation (5, p. 301)

$$\begin{aligned} y &= \cos (pt - \gamma s) + \cos (p't - \gamma' s) \\ &= 2 \cos \pi \left\{ t(f - f') - s(\gamma/2 - \gamma'/2) \right\} \cos \pi \left\{ t(f + f') - s(\gamma/2 + \gamma'/2) \right\}, \end{aligned}$$

and when the difference between the two frequencies is small, the group velocity will be about twice that of the phase velocity. In the case of two waves being sent into the bar from both ends, the two oscillators may not entirely be in resonance, on account of the coupling produced by the rod, and velocities of transverse waves even higher than those of any longitudinal wave are then possible. With purely longitudinal waves, or with transverse waves of very high frequency, for which according to the general formula the velocity changes only slowly with frequency, such an effect does not appear. It is also absent in the case in which two waves of the same frequency, but slightly different velocity, travel in the same direction.

The velocity of the radial waves and their interaction with the flexural and longitudinal waves will be discussed in a subsequent article.

Acknowledgment

The author is indebted to Dr. R. W. Boyle for his continued interest in this problem.

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MECHANICAL VIBRATIONS IN TRANSMISSION LINES¹

By C. D. NIVEN²

Abstract

Various types of vibration are discussed with special emphasis on the large and uncommon type of vibration known as "galloping". The effect of an ice coating on the line was investigated and could not be proved to enhance vibrations artificially introduced into the line. A mathematical expression for the period when the cable sways in the wind was developed. This period was found to be approximately equal to the period of the "fundamental" of the span. After various experiments had been carried out, the conclusion was reached which attributed the cause of galloping to pulsating winds.

Introduction

If a telephone or transmission line be viewed by an observer from a position in which his eye is on the same level as the wire, ripples can sometimes be seen travelling along the wire; or again, if one stands at the base of a steel tower, the whole tower often seems to be rattling. Both these phenomena are of common occurrence, but there is also another type of vibration which is seen only on very rare occasions and is known by engineers as "dancing" or "galloping". On these occasions the span vibrates either as a whole, or in segments, with the formation of nodes and loops. This phenomenon is of course a menace to the line, because once large vibrations start, the cable may be torn off the insulators before the motion subsides.

A very large percentage of the reports on galloping describe a coating of ice on the cable and this fact has misled engineers into formulating theories attributing the cause of these large vibrations entirely to ice. If such theories were valid, it should be impossible for galloping to occur without a coating of ice on the cable. Archbold's paper (1) in which a large number of reports on the phenomenon are assembled, clearly shows however that ice cannot be the basic cause of galloping but is merely an auxiliary agent. These large vibrations remind one so forcibly of the resonance of a stretched string to its fundamental note, that the possibility of galloping being a resonance phenomenon suggests itself at once.

The occurrence of these large vibrations is so rare that they are not of very great importance from an economic standpoint, in spite of the fact that the destruction which they may cause on a single occasion may be large; on the other hand, the continual occurrence of small vibrations is a serious matter, because eventually fatigue develops in the cable at the points of support.

The object of the investigations described in this communication was to arrive at some theoretical conclusions which would help to explain the cause of the small and large vibrations; for without the aid of theory on which to base one's reasoning, it is difficult to find suitable ways of coping with the phenomena in question.

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The Available Data

General

Before discussing the mathematics of a stretched string, it might be advisable to state briefly some information which the writer has acquired from various reports—verbal and written. Some of these reports were much more reliable than others, but generally speaking they lead one to believe the following statements:—

(a) When "galloping" occurs, ice is usually, but not always, present on the cables.

(b) "Galloping" usually occurs on lines which cross level, open country and in which the spans are of equal length and the elevations of the points of suspension are the same.

(c) A moderate wind, say about 25 m. p. h., is the most favorable wind to start galloping; gales are unfavorable.

(d) Very slackly strung lines and very tightly strung lines are both unfavorable for galloping.

The following statements are rather more doubtful:—

(e) Galloping occurs only on long lines.

(f) A wind blowing at an angle to the line is more likely to cause galloping than one blowing directly across the line.

(g) The most usual time for galloping to occur is at sunrise.

A few facts about the periodic motions which can occur in a cable when stretched between two supports are summarized below.

(1) When wind passes over a cable or other object, eddies are formed alternately on either side and cause the object to vibrate. This vibration, which may be called for convenience the "eddy" vibration, is of high frequency and is probably responsible for the rattle heard in steel towers, although this has not been proved. The diagrams in a paper by Varney (3) show that these small, rapid vibrations are superimposed on vibrations of larger amplitude and smaller frequency. However, as neither the frequency nor the amplitude of these eddy vibrations seems very suitable for building up large vibrations, they may be ruled out from the start as a possible cause of galloping. Nevertheless, as they are so frequently present, they cannot be entirely overlooked as a possible cause of the gradual fatigue found in cables at the points of support.

(2) When wind starts a cable swinging from side to side, there is set up a periodic motion very similar to that of a pendulum. This motion will be referred to as the "swing".

(3) When a disturbance is started in a span, both ends of which are fixed, as by striking the cable with a hammer close to one end, this disturbance travels to the other end, is reflected, and thus continues travelling to and fro. The frequency depends on the distance between the supports, the mass per unit length of the wire, and the tension, and is known in the theory of sound as the frequency of the fundamental note. It will therefore be referred to as such. It is of course identical with the resonance frequency.

These three periodic motions, giving rise respectively to the eddy vibration, the swing and the fundamental note, appear to be the only characteristic vibrations associated with a span.

There may, however, be other vibrations travelling over a span and arising from various causes, for instance from the swinging of neighboring spans, or from any periodic variation of tension set up in the line by vibration from, say, a factory or power house wall to which the line is attached, or from the vibration of the poles or insulators. A most interesting case of galloping observed in a 90-ft. span crossing a railroad is cited by Archbold (1). The suggestion is that a train passing by had been travelling at a rate such that the ground around the supports vibrated in resonance with the span. The writer has produced vibration in a model span by stepping heavily on the floor—around where the wire was supported—in time with the resonance vibration of the span.

Mathematical

Considerable attention has been given to the eddy vibration by Varney (3). His attention, however, seems always to have been focused on the eddies off the cable itself, no thought being given to the eddies off the pole or, in the case of tower lines, off the chain of insulators. Obviously there must be a movement, longitudinal so far as the line is concerned, arising from the flow of air past a cylindrical body as large as a pole or a chain of insulators.

According to Varney (3), the eddy frequency for a cable is given by the equation $f = \frac{v}{D} \times 0.185$. The diameter of the cable used by Varney was one inch; therefore D , the diameter, can be put equal to $\frac{1}{12}$. The frequency, f , is thus proportional to the velocity, v , of the wind. If now we consider the frequency of vibration of a nine-inch pole instead of that of the cable, the frequencies for different winds should be $\frac{1}{9}$ of those which Varney finds for the eddy frequency of the cable. Diagrams Nos. 2, 4, 5, 7, 14 in Varney's paper show respectively the presence of frequencies $\frac{1}{7}$, $\frac{1}{10}$, $\frac{1}{7}$, $\frac{1}{8}$ and $\frac{1}{10}$ that of the cable eddy. Had the cable been strung on nine-inch poles, instead of on towers, there seems to be no reason why the motion of the poles arising from eddies should not have increased the amplitude of these vibrations. At least it can be said that the eddy frequency around the poles or insulators seems much more likely to cause large vibrations than the eddy frequency around the cable itself.

The frequency of the swing can be calculated if it is assumed, first, that the cable hangs in a catenary and that the parabolic equation applies, and second, that the wire can be treated as a rigid body. With these assumptions the radius of gyration of the span about the line joining the points of support may be calculated. The equation for the cable is given by $y = \frac{x^2}{2c}$ where c is equal to the tension divided by the weight per unit length, and y is the distance from the directrix. If h be the height of the line joining the points of suspension above the directrix, then by putting $\eta = h - y$ the equation may be transformed to axes through a point midway between the points of support. The transformed equation is in fact $\eta = h - \frac{x^2}{2c}$.

Making use of the assumption that the catenary swings as a whole, like a rigid body, the radius of gyration, k , is given by

$$k^2 = \frac{\int \eta^2 ds}{\int ds} = \frac{\int \left(h^2 - \frac{hx^2}{c} + \frac{x^4}{4c^2} \right) ds}{\int ds},$$

where ds is an element of cable length. Assuming that the curvature is almost zero, we may put $x=s$, and integrate from 0 to x_0 where $2x_0$ is the distance between the points of support. Hence,

$$k^2 = \frac{h^2 x_0 - \frac{hx_0^3}{3c} + \frac{x_0^5}{20c^2}}{x_0},$$

and since $\frac{x_0^2}{2c} = h$, because $y=0$ at points of support, $k^2 = \frac{8h^2}{15}$, so that $k = 0.73 h$.

Therefore the period of swing $= 2\pi \sqrt{\frac{k}{g}} = 2\pi \sqrt{\frac{0.73h}{g}}$.

The period of the fundamental can be taken from the theory of sound, if, as has been assumed, the curvature is negligible; this practically amounts to assuming that gravity is small compared to the tension in the line. Under

these conditions the period of the fundamental may be written as $2s \sqrt{\frac{m}{T}}$ where m equals the mass per unit length and T equals the tension.

Now $2s \sqrt{\frac{m}{T}} = 2s \sqrt{\frac{W}{Tg}}$ where W is the weight per unit length. Therefore the period of the fundamental

$$= 2s \sqrt{\frac{1}{cg}}$$

where c is the constant which occurs in the equation for the catenary

$$\begin{aligned} &= 2 \sqrt{8ch \times \frac{1}{cg}} \\ &= 4 \sqrt{\frac{2h}{g}}. \end{aligned}$$

Referring back to the expression for the period of the swing, we have

$$\frac{\text{Period of swing}}{\text{Period of fundamental}} = \frac{2\pi \sqrt{.73}}{4 \sqrt{2}} = \frac{5.44}{5.66},$$

i.e. these periods are nearly the same.

The two periods are so close to each other that it was decided to arrange an experiment to ascertain if they were actually as close as theory predicted. With the aid of a stopwatch the oscillations per min. of the swing and of the fundamental were observed. They appeared to be identical. This experiment, in contrast to what usually occurs in physics, established a relation with greater accuracy than theory had indicated.

The Possible Effects of an Ice Coating on the Cable

As mentioned above, the reports indicated that a coating of ice facilitated the occurrence of galloping. Therefore, if it could be ascertained what effect ice had on a cable, it was thought that the clue to the cause of galloping would be found. Obviously, ice on the cable acts to some extent as a sail on a ship, in that it increases the total wind force on the cable without proportionally increasing the mass. The ice also forms an elastic coating on the outside of the cable and might therefore change the elastic constants. Then again under certain conditions, the ice might form on the cable as a wing and as Davison (2) has suggested, act as an airfoil. As such an ice formation is the exception rather than the rule, and as the lift forces which arise thereby are of necessity very small, the airfoil idea need not be discussed as a likely fundamental cause of galloping.

In order to investigate the possibility of a change in the elastic constants of the cable, of sufficient magnitude to cause the ice-coated cable to vibrate under conditions such that a bare cable would not vibrate, the following experiment was arranged:

By means of an electric motor, variable speed gear and eccentric, vibrations or rather variations of tension were introduced into 40-ft. span of copper wire 0.8 mm. in diameter. The variable speed gear made it possible for these vibrations to be of any frequency within certain limits, and it was therefore possible to plot the amplitude of vibration of the cable against the frequency of the exciting vibration. In this way, the resonance frequency could be found, because at that frequency the amplitude reached a sharp maximum, *i.e.*, galloping was in a sense artificially produced.

The experiment was then repeated out of doors during the winter months, in order to take advantage of the cold weather for the production of the ice-coating. A common syringe was used to make a fine spray of water resembling rain, and the ice-coating gradually accumulated on the wire where the spray struck it. The wire thus coated was then subjected to periodic variations of

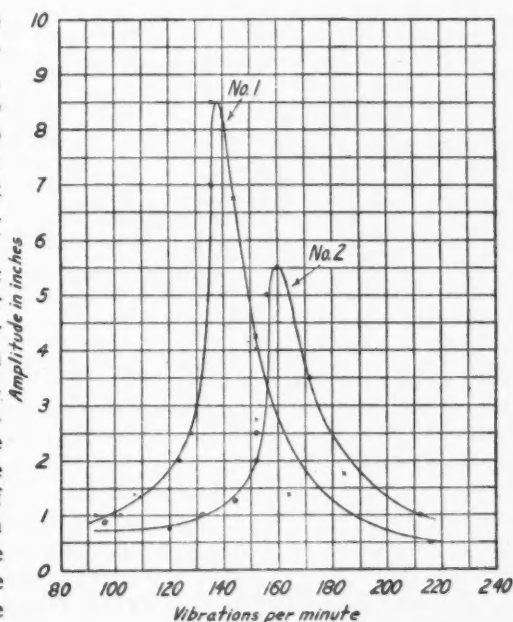


FIG. 1. Curves showing the variation in the amplitude of vibration of mid-point of span with variation of frequency of exciting vibration. No. 1—with no ice coating. No. 2—with ice coating.

tension or vibrations as before, and the amplitude plotted against the frequency. The energy of the exciting vibration was kept the same as before. The maximum amplitude when the ice was on the wire was found to be decidedly smaller than when the wire was bare, and there was no indication of tuning over a wider range of frequencies when the ice was on the wire. The ice-coated wire displayed a tendency to swing when in tune rather than to vibrate like a violin string.

In fact the experiment gave very definitely negative results, and by thus indicating that the ice-coating did not change the elastic constants in such a way as to cause galloping, it supported the other hypothesis, namely, that ice increased the effective wind force on the cable, without proportionally increasing the weight.

Fig. 1 illustrates the tuning of the exciting vibration to the resonance vibration of the span, first without any ice-coating and second with an ice-coating on the wire.

The Effect of Wind on a Span

The hypothesis that ice on the cable acts as a sail necessitated a thorough investigation of the possible effects which wind might have on the production of galloping. Wind can undoubtedly start the spans in a line swinging from side to side, and a motion thus started would theoretically cause a periodic variation of tension and so bring about a vertical motion in a neighboring span. To verify this experimentally, a three-span line was set up and the motor, referred to above, was arranged so that it gave one span a transverse, horizontal, periodic motion. The vertical motion could be observed in the other two spans, but it was much too small to account for galloping.

This result pointed to the probability that something intrinsically connected with wind was being overlooked. Therefore, merely with a view to getting an idea of what this omission might be, a wire was stretched between the legs of a table and the air stream from a common ventilating fan was directed upon it. Under these conditions the wire was found to vibrate. However, a ventilating fan gives a very turbulent air flow, so that there were undoubtedly pulses in the air stream, which were possibly the cause of the vibration. If this were so, a wire stretched in the carefully combed air stream of a wind tunnel should not vibrate; on the other hand, if the eddies referred to at the beginning of this communication, were in any way the cause of the vibration, the wire should vibrate in a wind tunnel just as readily as in front of a ventilating fan.

To settle this point, it was merely necessary to expose a stretched wire in the air stream of a wind tunnel and to ascertain whether it vibrated. In carrying out the experiment two wires were exposed to the wind. One of the wires was bare and the other was thickly coated with shellac, the latter representing ice. The air stream in the wind tunnel at the National Research Laboratories passes through a "honeycomb" grid which effectively eliminates turbulence. When the air stream flowed past the wires they bowed, but the vibration which had been observed when the wire was in the turbulent air stream from the ventilating fan did not occur. Wind velocities ranging from 20 to 60 m.p.h. were used in the experiment.

This result altered the complexion of the whole problem, for it seemed evident that it was a particular kind of wind which was responsible for the phenomenon of galloping. If this were the case, it was quite probable that the ice coating on the cable had no more to do with galloping than the fact that ice-coated cables were often associated with certain weather conditions, and these particular weather conditions were also associated with gusty winds of a particular nature. If, however, it were found that the vibration of the wire, when situated in front of a ventilating fan, increased as the shellac coating was built up, then there was evidence that in addition to the necessary wind conditions sometimes associated with sleet storms, the ice coating itself favored galloping. Accordingly the following experiment was arranged to ascertain the effect of an ice coating when the span was subjected to the action of a turbulent air stream. A three-span line was set up, the wire used being copper, 0.32 mm. diameter. The distance between the two end supports was 20 ft., and the three spans were all of the same length. A ventilating fan was placed opposite each of the three spans and the vertical motion was observed on a vertical scale placed at the middle of each span. The tension was adjusted to five ounces before the wire was finally fixed to the end supports. When the wire was bare the maximum vertical motion which could be observed was $\frac{1}{16}$ in., but after four coats of shellac had been put on, the maximum motion increased to $\frac{1}{8}$ in., and after ten coats to as much as $\frac{5}{16}$ in. The thickness of the wire was then about 1.25 mm. Clearly the shellac coating was increasing the vertical vibratory motion. It was also found that when the wire was not free to move in a longitudinal direction at the middle supports, the maximum amplitude with ten coats of shellac decreased to $\frac{3}{16}$ in. The experiment clearly indicated, first, that a coat of ice on a cable would increase any vibration caused by a gusty wind, and second, that a vibration in a span could be reinforced by vibrations from other spans, and that a long line would therefore favor the building up of a large vibration and the consequent appearance of galloping.

Summary and Conclusions

A periodic variation of tension, if it is of period identical with that of the fundamental of the span, will cause the span to gallop. In the laboratory this was produced by a motor and eccentric, but it might perhaps be produced in an actual line by vibration from a factory wall, or the vibration of the supports, caused either by air eddies from the poles or insulators, or by ground vibrations.

By virtue of the fact that the swing of a span has the same frequency as the fundamental, the swinging of spans may cause vibrations in a line in which the lengths of the spans are equal, but these vibrations are too small to account for galloping.

Ice does not increase the amplitude of vibration caused by a periodic variation of tension which may have started in the line, say through the agency of a vibrating power-house wall, but rather decreases it. Judging from the fact that in the majority of the cases of galloping, an ice coating has been reported on the cable, one might be led to conclude that ice itself causes galloping. This

seems unlikely unless ice acts as a sort of sail on the cable, but this conception implies certain kinds of gusty winds. Therefore, if suitable gusty winds must be assumed, there may be no necessity for an ice-coating at all. A thick ice coating increases the vibration caused by a gusty wind but it seems probable that the most important agent is the suitable gusty wind. As sleet is often accompanied by peculiar weather conditions, the conclusion is that the cases of galloping which writers on the subject have up to the present associated in some way with an ice coating, should really have been traced to the particular weather at the time—weather which was suitable to the simultaneous production of an ice coating on the cable and the requisite wind conditions. If such a conclusion be correct, the phenomenon of galloping is practically dependent on meteorological phenomena and further research on the subject should take into consideration particular types of wind associated with particular weather conditions. Before suggesting ways and means of preventing galloping these conclusions should be verified in the field and therefore suggestions are in a sense premature. It might, however, be possible to design some sort of air foil which could be attached to the line and which would turn in such a way, when the wind blows, that the vibratory motion would be damped. Of course, economic considerations would prevent the use of too expensive a remedy.

Acknowledgment

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A GAMMA RAY IONIZATION CHAMBER¹

BY GEORGE C. LAURENCE²

Abstract

A description is given of a γ -ray ionization chamber with its accessory electrometer box, suitable for precision measurements of radioactive preparations, and for the measurement of the absorption coefficient and specific ionizing powers in air, of γ -rays.

This ionization chamber has been designed for precision measurements of γ -radiation in the National Research Laboratories at Ottawa, where an instrument is required of sufficient flexibility to serve several purposes. It is being used for the recalibration measurements of the national radium standards of 1, 2, 5, 10 and 25 mgm. These are being compared with similar calibrations made with a gold-leaf electroscope in a search for consistent discrepancies, since the difference in method may tend to expose instrumental errors peculiar to either. It is intended for measurements of γ -ray absorption coefficients for the estimation of mesothorium impurity, for the determination of γ -ray ionization of air per unit volume, and finally, it is used for the measurement of radioactive preparations submitted to the Council for calibration and for the estimation of their mesothorium impurity. For these purposes its shape is more suitable than the usual gold-leaf electroscope. It has proved very satisfactory, yielding higher precision than is usual with the gold-leaf electroscope. This advantage is probably due to the use of a null method and long effective scale length as described below.

The ionization chamber, supported in its mounting, may be seen in Fig. 1. (Letter references in the text correspond to the wiring diagram, Fig. 2.) The radium capsule is carried in the V-shaped bucket on the left. This has a thin aluminium wall on a rigid brass frame which fits snugly in place on the top of the support, which may be clamped to the steel ferrule at the desired distance from the chamber.

Removal of the cover, visible on the back of the chamber, gives access to the collecting plate, *c*, which is of brass, 10 cm. square, and occupies a somewhat larger hole in a brass plate, *g*, 30 cm. square, which acts as a guard ring, forming a border 10 cm. wide around the collecting plate. The guard ring also forms the supporting wall of the instrument. It is bolted to the iron supporting frame, carries the box on the back, and holds the collecting plate with four quartz insulators. The face of the chamber, *d*, an aluminium plate 30 cm. square, is fastened at a suitable distance in front of the collecting plate and guard ring, on an ebonite separator, which in turn is fastened to the guard ring. This plate and separator are readily removed, so that separators of different thicknesses may be inserted, thin foils of absorption material may be put on the electrodes to reduce photo-electric emission from them, and lead absorption screens may be added as desired. The brass cup on the bottom of the box holds

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a small beaker of desiccating material, and may be removed for cleaning by unscrewing. A manometer tube, hidden behind the box, Fig. 1, indicates the pressure in the chamber. In practice the cracks around the cover and elsewhere are sealed with soft wax. A block of paraffin wax is cast on the back of the collecting plate and around the connecting wire leading from it to its terminal on the left side of the box. This reduces the flow of ions to the collecting plate from the air space behind it. The connection to the electrometer is carried through the evacuated brass tube on the right.

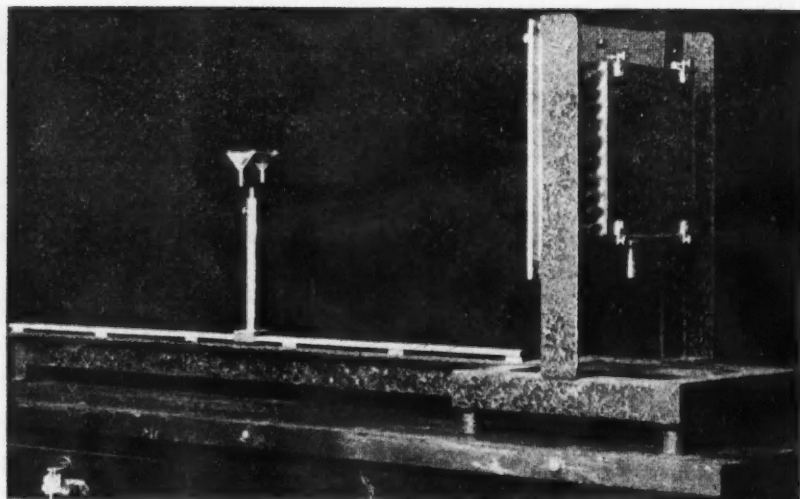


FIG. 1. *Gamma ray ionization chamber.*

The Lindemann electrometer used with the chamber is enclosed in a brass box which is evacuated by a rotary oil pump. The important details of the box are shown in Fig. 2. The reading telescope is mounted directly on the cover and a microscope mirror is supported underneath. The base is of wood and is screwed to the table. One of the four brass supporting legs is hollow and serves as an outlet for pumping out the air and the connecting leads to the electrometer are brought out through a glass pinch waxed into it. A condenser, *C*, of the coaxial cylinder type (capacity about 100 e.s.u.) is built into the tube which shields the connection to the ionization chamber. The earthing switch, *e*, is soldered into a short length of flexible corrugated metal tubing (not shown) so that it can be operated from the outside of the box. The end of the switch is shown in the drawing protruding into the chamber through its cylindrical wall. Motion in the direction indicated by the arrow brings the pin into contact with a phosphor bronze spring on the electrometer terminal (broken line) thereby earthing it.

The usual null method commonly called "tram driving" is used. The flow of ions to the collector *c* charges the inside cylinder of the condenser *C*, which

is kept close to earth potential during a measurement, by moving the rheostat R' which alters the potential of the outside cylinder of C . Readings are timed from the passage of a convenient division (near zero) in the electrometer scale with R' in the position a ; to the passage of the same division with R' in the position b . Thus the collecting electrode and connection can be kept to within 0.01 volts of earth and guard-ring potential during a reading.

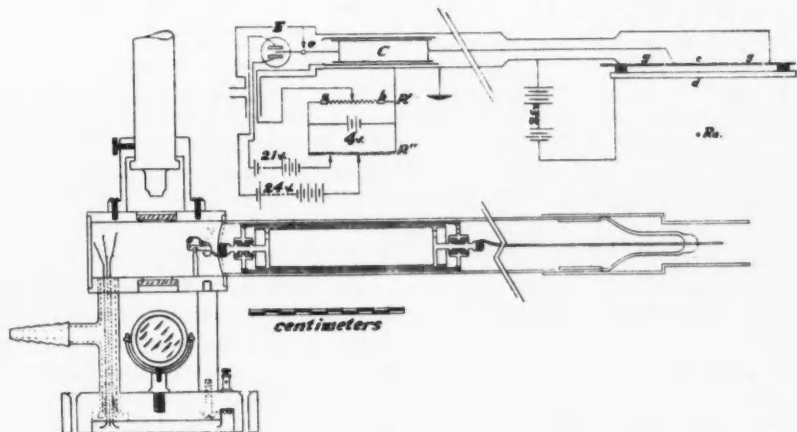


FIG. 2. Lindemann electrometer box, and diagram of connections.

The most serious sources of error in using this type of instrument may arise from the flow of ions in the electrometer, the condenser, around the connecting wire and behind the collecting plate, and from the inertia of the electrometer needle. The former is reduced by evacuation of the box containing the electrometer condenser and connection wire, and by waxing the back of the collector as described, and with care in keeping the collector system close to earth potential during measurements, it can be kept under 0.1%. The latter is kept insignificant by choosing a timing division which is a short distance on that side of the zero division to which the needle tends to move as the collector charges up, but, of course, not far enough from the zero division to introduce error from the other cause. Effects due to slow changes in conditions, such as the voltages of the electrometer batteries, are dealt with by bracketing readings.

The sensitivity in scale divisions per second for a given radiation intensity is about the same as that of a good gold-leaf electroscope; for example, a deviation of 1 division per sec. is obtained with 25 mgm. of radium at a distance of 50 cm. from the instrument. Advantage is taken of the long effective scale length made available by increasing the voltage of the condenser-charging battery, so that in practice, fewer readings of longer duration are taken than with a gold-leaf electroscope. Four volts giving an effective scale length of about 500 divisions is convenient, but larger voltages can, of course, be used. Long series of readings with the instrument have shown average departures from the mean of $1\frac{1}{2}$ per 1000.

REVIEWS AND NOTES

IMPROVEMENTS IN THE ANALYSIS OF
MAPLE PRODUCTS¹BY J. F. SNELL², LEV SKAZIN³, H. J. ATKINSON³ AND G. H. FINDLAY³

Abstract

Collaborative work on maple syrup has shown that, with refractometric control, samples can be prepared to a content of 65% solids with much greater precision than is attainable when boiling temperature alone is depended upon. As the dry basic lead acetates used in different laboratories vary greatly in solubility and alkalinity, much better concordance in Canadian lead values can be attained by use of a reagent prepared from normal lead acetate and litharge "activated" at 650-670° C. The electrical conductivity of solutions containing 25% of solids is less variable in genuine goods than any of the chemical values. The soluble ash is less variable than the insoluble.

Water Content and Preparation of Sample

Excess of water in maple syrup and sugar is objectionable not merely as representing weight without value but also because it renders the syrup more susceptible to deterioration by micro-organisms, and the sugar soft and less coherent and therefore less convenient to handle. The regulations under the Food and Drugs Act (1a) require that the water content shall not exceed 10% in the sugar, 35% in the syrup or 15% in maple butter, cream or wax. In syrup a content of water *lower* than about 32% is apt to cause sucrose crystals to separate, an effect that is commercially objectionable.

In the examination of maple products for adulteration it is usual to dilute, reboil and filter syrups and to convert solid products into filtered syrups, thus bringing into solution any soluble substances that have separated out as the result of evaporation beyond the standard syrup stage, and removing any that have remained in solution on account of under-evaporation. The aim, of course, is to bring the syrup as near as possible to the standard water content of 35% (1a, 6). As the analysis is then made with reference to this "prepared" syrup, a second measurement of water content is necessary. The determination of water (or total solids) in syrups is commonly made with an Abbé type of refractometer. It is obviously desirable that the directions for this determination should be such as will lead to consistent results in the hands of different analysts and also that the directions for preparing the sample for analysis should be so stated as to enable them to obtain products of equal water content.

In the preparation of the sample in the year 1929 the collaborators followed the directions of the 1925 edition of the A.O.A.C. Book of Methods (1), *viz.*, boiling the diluted sample to a temperature of 104° C. and filtering through

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³ Research Assistants.

cotton wool. Amongst 10 collaborators, none succeeded in preparing 20 syrups with a water content showing less variation than 5.8%. The average difference between the highest and lowest percentage of water in syrups prepared by each analyst was 7.7. Only two of the nine analysts who adhered to the directions prepared their syrups so that the average solids content fell within 1.3% of the standard 65%, the prepared syrups of others averaging from 57.4 to 63.7%.

As each collaborator had made his own refractometric determination of water it was not clear how much the variations in this determination in the hands of the different analysts had contributed to the differences shown in the results of preparing the syrups. In 1930, therefore, not only was an improved method of preparation of the sample proposed but analysts were also requested to report total solids in the samples as received as well as total solids after preparation. The results revealed a surprisingly wide variation in the refractometric results obtained by different analysts with identical syrups. On only half of the 30 samples examined did all collaborators report total solids in agreement within 1.0% solids. In one case the difference between the highest and lowest values reported was a little over 5%, in five between 2 and 5% and in nine between 1 and 2%.

Experiments in the Macdonald College laboratory indicated that the reading on the thermometer of the Abbé refractometer (upon which reading a correction to 20° is based) might not truly represent the temperature of the film of syrup spread between the prisms. By comparison of readings made upon a few syrups by several observers it was established that more consistent results were obtained when water was circulated through the jacket of the instrument during the observations. To obviate the possibility of dew deposition it was deemed advisable to use water of room temperature and to correct for temperature, rather than to attempt to make all readings at 20° C.

With the observance of this precaution in 1931 much more concordant collaborative results were obtained than in 1930. Amongst reports of 13 analysts on 8 syrups (104 reports) only a single one, or 1%, differed from the mean by more than 0.75% as against 7% (17 in 234) in 1930. The average deviation from the mean was 0.252% of solids, as against 0.354% in 1930.

The revised method of preparation required that when the temperature of the boiling syrup approached 104° C. the analyst should from time to time withdraw small samples, cool them and examine them in the refractometer. As a filtering medium extra rapid filter papers were substituted for the cotton wool. Two brands of paper were found which would filter 100 cc. of syrup in less than five minutes. The general average of total solids in syrups so prepared was 65.6% in 164 preparations in 1930 and 65.3% in 88 preparations in 1931, as against 61.3% in 202 preparations by the old method in 1929. The number of analysts who succeeded in preparing all syrups within a range of 1% was 1 amongst 8 in 1930 and 6 amongst 11 in 1931; within a range of 2%, 3 in 1930 and 10 in 1931; whereas in 1929 none amongst 10 had succeeded in keeping within a range of 5%. This modified method has accordingly been substituted for the old one by the Association of Official Agricultural Chemists.

Conductivity Value

The measurement of the electrical conductivity of maple syrup diluted to a definite concentration approximating that at which the conductivity attains its maximum was proposed in 1911 (12, 13, 14) as a rapid test for adulteration with refined sugar and was adopted by the Association of Official Agricultural Chemists as a Tentative Method in 1919 (7). Experience has demonstrated that the value determined by this method shows less variation in genuine syrups than any of the chemical values (15, 18, 19) and though the directions as given in the A.O.A.C. Book of Methods can probably be modified to advantage (particularly with reference to the determination of the "cell constant") the method is proving of value and will no doubt find more general use now that conductivity apparatus suitable for this purpose is being used in the sugar industry (11, 22).

First action towards the promotion of this method to the status of an Official Method was taken by the Association of Official Agricultural Chemists in 1930 (8), but final action is deferred pending revision of the directions. The solution now used is that containing 25 gm. of dry matter per 100 cc., instead of that containing 22 gm. originally recommended.

Canadian Lead Value

The measurement of the weight of the precipitate produced by basic lead acetate in solutions of maple sugar under specified conditions was proposed by the Laboratory of the Inland Revenue Department* in 1906 (9, 10) and adopted as a Tentative Method by the Association of Official Agricultural Chemists in 1919 (7). In 1928 the method was studied in detail by Fowler and Snell (2) and modified by the substitution of cold for hot water in the washing of the precipitate. Collaborative studies of this method in both forms and with the use of smaller quantities of the reagent have led to the adoption (8) of directions conforming to the Fowler modification.

The 1930 collaborative work revealed wide variations in the basic lead acetate solutions prepared in different laboratories (19) from dry lead subacetate. The alkalinities of such solutions were found to range from 6.51 to 10.14 (cc. 0.1 N per cc. solution), pH values from 7.1 to 7.5 and total lead per cc. from 0.1991 to 0.2397 gm. In 1931 (19a), a new method of preparing the reagent was proposed, *viz.*, activating litharge by heating to 650-670° C. for 2½ to 3 hr., and dissolving in boiling water (250 cc.) the lemon-yellow product (40 gm.) and normal lead acetate crystals (80 gm.) in the proportions used in preparing basic lead acetate solution before the dry basic acetate became a common chemical. Collaborators sent to Macdonald College not only the reagents prepared in the two ways but also portions of the solid chemicals used in their preparation.

* Mr. Thomas Macfarlane was Chief Analyst in 1906 and the method was originated at his suggestion by Mr. A. Valin, now analyst in charge at the Montreal branch of the Food and Drugs Laboratory of the Department of Health, successor to the Laboratory of the Inland Revenue Department.

Solutions prepared from these materials by a single chemist (G.H.F., who activated all the samples of litharge and also made the analyses of the collaborators' reagents as well as of those prepared by him) did not vary much from those made in the collaborating laboratories. Amongst 10 samples of dry subacetate, the greatest differences between any of the solutions prepared by collaborators and those prepared by G.H.F. were: alkalinity, 0.35 cc., 0.1 N; density, 0.008; and total lead, 16.1 mgm. per cc. The average differences without regard to sign were: alkalinity, 0.14 cc.; density, 0.0035; and total lead, 7.2 mgm. per cc. The range of alkalinity was slightly narrowed, from one of 5.23-11.33 to one of 5.39-11.25; the pH range (7.1-7.6) was unaffected. A solution kept over from 1930 by one of the collaborators showed somewhat greater variations from that prepared (in 1931) from the same solid by G.H.F., *viz.*, +0.62 in alkalinity, +0.014 in density and +14.6 mgm. per cc. in total lead.

Amongst nine pairs of solutions prepared by collaborators and G.H.F. from the nine samples of litharge the only differences in alkalinity greater than 0.30 were two instances in which the collaborating analysts had obtained solutions having alkalinities of 9.80 and 9.83. With these two exceptions all the collaborators' solutions fell within the range 10.11-10.51, which was practically identical with that for the solutions prepared by G.H.F., *viz.*, 10.21-10.57. The collaborators' solutions showed pH values of 7.5 to 7.6, those made by G.H.F. from 7.4 to 7.6. The total lead per cc. showed remarkably little variation in either the collaborators' or G.H.F.'s preparations (0.2316-0.2385 in the former, 0.2254-0.2328 in the latter) but was always somewhat lower in the latter.

Comparing the reagents prepared in the two ways, it was found that those made by the new method showed much less variation in alkalinity and a higher general plane of alkalinity than those made from the dry basic acetates. (In the collaborator's preparations, the average alkalinity was 10.24 for the new reagents, as compared with 8.17 for the old). With practically all the syrups they also gave higher lead numbers than were yielded by the reagents prepared from dry basic acetate. The exceptions were among the results of (a) one analyst whose dry basic acetate solution had an exceptionally high alkalinity (11.33), actually exceeding that of his activated litharge reagent, and (b) a second analyst whose litharge reagent had an alkalinity that was the lowest of its class (9.80), the alkalinity differing but little from that of his dry basic acetate reagent (9.29). The other analyst with a litharge of low alkalinity (9.83) happened to have the least alkaline of the dry subacetate reagents (5.23), so that in this case the difference between the alkalinities of the two reagents was one of the greatest found, and the differences between his lead values with the two reagents were correspondingly large.

That there is a close correlation between alkalinity of reagent and magnitude of lead value is further shown by the fact that the magnitudes of the various analysts' average differences between lead values obtained with the two reagents in the eight syrups run in the same order as the differences between the alkalinities of the two reagents. This was further corroborated by experiments which showed (a) that lower lead values were obtained by use of a

lead subacetate solution diluted with acetic acid than by use of the same solution equally diluted with water, and (b) that a series of five solutions of equal specific gravity, made with varying proportions of activated litharge and normal acetate so as to show alkalinity varying from 12.16 to 2.72, (ratios of neutral to basic lead 0.83 to 6.68) gave lead values decreasing steadily in one syrup from 4.52 to 2.12 and in another from 3.69 to 1.80.

The lead values reported for each syrup by the various collaborators were also more concordant with the reagent prepared from activated litharge than with that made from the dry salts. The average range of difference for the litharge solution was 13.2% of the average value as against 30.9% of the average with the reagents prepared from the dry salts, while the maximum range was 27.1% as against 41.5% of the average.

The ranges of the lead values found by all but two of the analysts for the eight syrups were, however, somewhat wider with the new than with the old reagent. The average range for all the analysts with the new reagent was 53.2% of the average value as against 49.7% with the old. Whether this will remain true in a larger and more varied collection of syrups remains to be determined, as does also the question whether the advantageous rapid decrease of the old lead values upon progressive dilution with refined sugar syrup (20) will characterize values determined with the new reagent. Further it is possible that a reagent with other proportions of basic and neutral lead might prove better than that in which the proportions formerly employed with ordinary litharge have been used.

Winton Lead Values

The Winton method (21), proposed practically simultaneously with the Canadian method, differs from the latter both in the use of a more dilute reagent and in the expression of results in terms of lead precipitated instead of the weight of the precipitate as a whole. Winton values are, of course, lower than the Canadian values for the same syrups, and apparently fall off proportionally to, rather than more rapidly than, the percentage of maple sugar in mixtures with refined sugar (20). The effect of modification of the method of preparation of the reagent along lines similar to those followed with the Canadian method might very well repay study.

Ash Values

In the 1929 collaboration wide variations were found in the values of total, soluble and insoluble ash, and in the alkalinities of the soluble and insoluble ash reported by collaborators. The least variable of these values appear to be the alkalinities and percentages of the soluble and of the total ash as determined upon the prepared syrups. Among 20 syrups the ranges of these values were narrower than those of the Fowler lead value and the insoluble ash values, though greater than that of the conductivity value. Further studies on these values have not been undertaken.

For further details of these studies of analytical methods, the reader is referred to the reports made to the Association of Official Agricultural Chemists and published in the Journal of that Association.

Summary

1. The outcome of work upon methods of analysis of maple syrup and sugar carried on over a period of years in collaboration with other Canadian and American chemists is summarized.

2. In the refractometric determination of solids in syrup, it is of advantage to circulate water of room temperature through the jacket of the Abbé type instrument.

3. The preparation of samples to a content of 65% of solids can be accomplished with much greater precision when observation of boiling temperature is supplemented by refractometric control.

4. For filtration the use of such filter papers as will filter 100 cc. of hot syrup in less than five minutes is recommended.

5. The electrical conductivity of solutions of maple sugar containing 25% of solids is less variable in genuine goods than any of the chemical values.

6. The dry basic lead acetates used in various laboratories vary greatly in solubility and alkalinity.

7. Reagents of much greater uniformity can be prepared from litharge "activated" at 650-670° C. and normal lead acetate. These give more concordant Canadian lead values than the solutions prepared from the dry basic acetates.

8. Of the ash values, those showing least variation in genuine goods are the amount and the alkalinity of the soluble ash.

Acknowledgment

The thanks of the authors are due to a number of chemists* in Canada and the United States for loyal and continued collaboration.

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THE MECHANISM OF POLYMERIZATION REACTIONS¹

BY WILLIAM CHALMERS²

Abstract

Attention is called to an earlier, unpublished writing by the author wherein a chain-reaction mechanism was suggested for all polymerizations leading to macro-molecular products. It is further pointed out that the only scheme of reaction which is compatible with this mechanism is that which involves only the double bond, *i.e.*, the possibility of the changes taking place by the transference of hydrogen atoms is practically excluded.

The appearance of papers by Milas (3), Starkweather and G. B. Taylor (4) and, more recently, by Hugh S. Taylor and Vernon (5) and by Conant and Peterson (2) seems certain to establish a chain mechanism in several polymerization reactions. That all polymerizations of olefinic compounds leading to the formation of macro-molecular bodies take place by a chain reaction was proposed by the writer in a report to the National Research Council in February 1930 (1, pp. 32-33). This study was based upon semi-quantitative work done by various chemists and will be elaborated in a paper which will appear in the near future. The following brief account differs only slightly in mode of presentation from that given in a section on the "Kinetics of Polymerization" in the report mentioned.

The key to the nature of the reactions is given by the observation that only traces, if any, of lower polymers are formed. Particularly is this apparent in the photo-polymerization of the vinyl halides and vinyl cyanide where the deposition of polymer commences almost immediately on exposure to the activating radiation, although the residual monomer does not alter appreciably in properties. A reaction of the following type is clearly indicated:

$\text{CH}_2:\text{CR}_1\text{R}_2 \xrightarrow{*} \text{CH}_2:\text{CR}_1\text{R}_2 \text{ (activated)}$ —at a rate comparable with those of the ordinary reactions of organic compounds—

$\text{CH}_2:\text{CR}_1\text{R}_2 \text{ (activated)} + (n-1) \text{CH}_2:\text{CR}_1\text{R}_2 \longrightarrow (-\text{CH}_2-\text{CR}_1\text{R}_2-)_n \text{ (polymer)}$ —practically instantaneous—.

The application of this chain mechanism is not confined to photo-polymerization but applies to similar changes under the influence of heat and catalysts.

In the simplest case of such a reaction, a certain fixed proportion of the molecules becomes activated at any moment. That is to say, the rate of formation is proportional to the concentration of monomer. The velocity of activation is

* $\text{CH}_2:\text{CR}_1\text{R}_2$ is adopted as a general formula for olefinic compounds showing a tendency to transformation to high-molecular polymeric forms, R_1 representing an unsaturated (negative) group or a halogen atom and R_2 a hydrogen atom, alkyl group, or another "negative" substituent.

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² Holder of scholarships under the National Research Council of Canada, 1927-1931, at McGill University, Montreal, Canada, and the University of Freiburg, Germany.

given by the familiar unimolecular equation,

$$\left(\frac{dx}{dt}\right)_1 = k(a-x),$$

(where a is the initial concentration of monomer and x molecules of monomer are converted into the polymer at the end of time t). It seems to hold true, at least over a large range, that the (average) order of polymerization does not vary materially during the course of the reaction. If this factor be represented by n , the velocity of aggregation will be given by the expression:

$$\begin{aligned}\left(\frac{dx}{dt}\right)_2 &= n \cdot \left(\frac{dx}{dt}\right)_1 \\ &= kn(a-x) \\ &= K(a-x).\end{aligned}$$

Thus the reaction will be pseudo-unimolecular.

While in many instances complications are introduced by auto-catalysis, as well as by side reactions, at least several known cases approximate to such simple behavior.

With a knowledge of the actual nature of polymerization reactions we are enabled to proceed to an important conclusion which may be outlined briefly as follows. The formation of the macro-molecule takes place at a rate comparable with the ionic reactions of inorganic chemistry. Any theory, then, which pictures the polymerization of styrene and related compounds as taking place in distinct stages by the transference of hydrogen atoms is rendered highly unlikely. The only type of reaction which can be brought into accord with the observed kinetics is that in which the double bond (more accurately, the carbon atoms bound by the double linkage) alone takes part. This would require only the rearrangement of electrons and can be readily pictured as taking place with a speed far exceeding any of the ordinary reactions of organic chemistry where atomic transfer is involved.

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